

Indiana Collegiate Math Competition 2021

- Let $A = \begin{pmatrix} 1 & 2021 \\ 0 & 1 \end{pmatrix}$.
 - (5 points) Find A^{2021} .
 - (5 points) Find a 2×2 matrix B so that $B^{2021} = A$.
- The polynomial $p(x) = 16x^4 - 32x^3 - 104x^2 + 122x + 232$ has an interesting property. There is a line $y = mx + b$ that is tangent to the graph of p in *two* places. Find the line. Hint: consider the polynomial $q(x) = p(x) - (mx + b)$ and how it might factor.
- A three-digit positive integer n is exactly 5 times the product of its digits.
 - (5 points) Show that the digits of n must all be odd.
 - (5 points) Find n .
- Consider the points $A(3, 4, 1)$, $B(5, 2, 9)$ and $C(1, 6, 5)$ in R^3 . Show that these points are the vertices of a cube.
- Let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$. A partition P of S into two element subsets $\{\{x_1, y_1\}, \{x_2, y_2\}, \{x_3, y_3\}, \{x_4, y_4\}\}$ has each x_i and each y_i a value from S , with all values used precisely once.
 - (4 points) How many partitions of S into four two-element subsets are there?
 - (6 points) For a given partition, define its *value* by $V(P) = \sum_{i=1}^4 x_i y_i$.
We say an integer n is *achievable* if there is a partition P whose value $V(P)$ is n . Find the minimum and maximum achievable values and explain why they are the minimum and maximum (3 points each).
- Let \underline{xyz} be a three-digit number, made with digits x , y , and z . That is, $\underline{xyz} = 100x + 10y + z$.

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- (a) (5 points). Find digits a , b , and c , not necessarily distinct, for which $\underline{abc} + \underline{cab} - \underline{bca} = \underline{608}$.
- (b) (5 points). Show that there are no values of a , b , and c that satisfy $\underline{abc} + \underline{cab} - \underline{bca} = \underline{707}$.
7. Polynomial p has non-negative integer coefficients, and satisfies $p(1) = 21$ and $p(11) = 2021$. What is $p(10)$?
8. In a “KenKen” puzzle, the numbers in each heavily outlined set of squares, called cages, must combine (in any order) to produce the target number in the top corner of the cage using the mathematical operation indicated. A number can be repeated within a cage as long as it is not in the same row or column. In the 5×5 puzzle, each of the digits 1 through 5 must appear precisely once in each row and column. Solve the 5×5 KenKen below.

55, +	288, ×			