PROBLEM 1 TEAM #

Suppose that the points P and Q are randomly selected in the interval $[0, 2]$. What is $\Pr[|\overline{PQ}| \leq \frac{1}{3}$?

Show work to be graded below, and use the reverse side of the page to continue if necessary

PROBLEM 2 TEAM #

Show that the Maclaurin series of

$$f(x) = \frac{x}{1 - x - x^2}$$

is equal to $\sum_{n=1}^{\infty} f_n x^n$, where $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$.

Show work to be graded below, and use the reverse side of the page to continue if necessary

PROBLEM 3 TEAM #

Let $T = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Let the set S be defined as follows.

$$S = \{f | f : T \to T \text{ is a bijection}\}$$

with binary operation function composition. Let $\sigma \in S$. Suppose that σ^3 is defined as follows.

$$\begin{aligned}
\sigma^3(1) &= 2 \\
\sigma^3(2) &= 3 \\
\sigma^3(3) &= 5 \\
\sigma^3(4) &= 6 \\
\sigma^3(5) &= 7 \\
\sigma^3(6) &= 1 \\
\sigma^3(7) &= 8 \\
\sigma^3(8) &= 4
\end{aligned}$$

What is σ ?

Show work to be graded below, and use the reverse side of the page to continue if necessary

PROBLEM 4 TEAM #

Show that $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=1$.

Show work to be graded below, and use the reverse side of the page to continue if necessary

PROBLEM 5 TEAM #

Suppose that n is a composite number, $n > 0$ and $n \neq 4$. Show that $n | (n-1)!$.

Show work to be graded below, and use the reverse side of the page to continue if necessary

PROBLEM 6 TEAM #

Suppose that $a, b \in \mathbb{R}$, with $a < b$. Suppose that $f : (a, b) \to \mathbb{R}$. Suppose that f is increasing and satisfies the property that for all $\lambda \in (0, 1)$ and $x, y \in (a, b)$

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y)$$

Prove that f is continuous on (a, b).

Show work to be graded below, and use the reverse side of the page to continue if necessary

PROBLEM 7 TEAM #

Let $\mathbb{Z}_{\geq a}$ be equal to the set $\{x | x \in \mathbb{Z}, x \geq a\}$. It is known that there is a 1-1 correspondence $F : (\mathbb{Z}_{\geq 0})^{\times 3} \to \mathbb{Z}_{\geq 1}$. Find a formula for F.

Show work to be graded below, and use the reverse side of the page to continue if necessary