PROBLEM #1 TEAM #____

Let S be a set with a binary operation $*$ that is associative. Suppose that for all x and y in S we have $x * x * x = x$ (i.e. $x^3 = x$) and $x * x * y = y * x * x$ (i.e. $x^2 y = y x^2$). Show that for all x and y in S we have that $x * y = y * x$.

Write work to be graded for problem 1 below and continue on the reverse side, if necessary.

PROBLEM #2 TEAM #____

Two friends agree to meet at the library, but each has forgotten the time they were supposed to meet. Each remembers that they were supposed to meet sometime between 1:00 pm and 5:00 pm. They each independently decide to go to the library at a random time between 1:00 pm and 5:00 pm, wait for 30 minutes, and leave if the other doesn't show up. What is the probability that they meet during this 4-hour period?

Write work to be graded for problem 2 below and continue on the reverse side, if necessary.

PROBLEM #3 TEAM #____

Suppose that two triangles have a common angle. Show that the sum of the sines of the angles will be larger in that triangle where the difference of the remaining two angles is smaller.

HINT: $2\sin\left(\frac{\theta+\delta}{2}\right)\cos\left(\frac{\theta-\delta}{2}\right) = \sin\theta + \sin\delta$.

Write work to be graded for problem 3 below and continue on the reverse side, if necessary.

PROBLEM #4 TEAM #____

The base of a solid object is the region bounded by the parabola $y = \frac{1}{2}x^2$ and the line $y = 2$; cross sections of the object perpendicular to the y-axis are semicircles. What is the volume of the object?

Write work to be graded for problem 4 below and continue on the reverse side, if necessary.

PROBLEM #5 TEAM #____

Define the sequence $\{x_n\}_{n=0}^{\infty}$ by $x_0 = 0$, $x_1 = 1$, $x_n = \dfrac{x_{n-1} + (n-1)\,x_{n-2}}{n}$. Determine $\lim_{n\to\infty} x_n$. Recalling that

$$\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k\, x^{k+1}}{k+1},\ \ x \in (-1, 1]$$

may be helpful.

Write work to be graded for problem 5 below and continue on the reverse side, if necessary.

PROBLEM #6 TEAM #____

Prove that $\dfrac{1}{n+1}\dbinom{2n}{n}$ is an integer for all integers $n \geq 1$.

Write work to be graded for problem 6 below and continue on the reverse side, if necessary.

PROBLEM #7 TEAM #____

Find matrices B and C such that $B^3 + C^3 = \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}$.

Write work to be graded for problem 7 below and continue on the reverse side, if necessary.