ABSTRACTS OF TALKS PRESENTED TO THE INDIANA SECTION OF THE MAA

1. Introduction

The Fall 2023 meeting of the Indiana Section of the Mathematical Association of America is being held at the University of Notre Dame, October 7. The abstracts appearing here are based on text electronically submitted by the presenters.

Contributed presentations are listed in alphabetical order by presenter.

2. Invited Talks

Presenter: Dominic Klyve, Central Washington University, Editor of the College Mathematics Journal

Mathematical Fights! The Seedy Underbelly of Mathematical History

Although students are often led to believe that mathematics is a purely rational, unemotional, and orderly field of study, history shows that this is often not the case. This talk will discuss some of the greatest fights in the history of mathematics. We will hear stories of friendships destroyed and national rivalries heightened because of disagreements about underlying mathematics. We will consider what these fights teach us about the nature of mathematics, and we will learn some interesting math on the way.

Presenter: Juanita Pinzón Caicedo, University of Notre Dame

Joint work with: Allison Miller, Swarthmore College, and Peter Feller, ETH Zürich

Comparing the Topological and the Smooth 4*-genus of Satellite Knots.*

The study of 4-dimensional objects is special: a manifold can admit infinitely many non-equivalent smooth structures, and manifolds can be homeomorphic but not diffeomorphic. This difference between topological and smooth structures, can be addressed in terms of the study of knots as boundaries of surfaces embedded in 4D space. In this talk I will focus on some knot operators known as satellites and will show that satellites can bound very different surfaces in the smooth and topological category.

3. Indiana Project NExT Panel Discussion

Panelists:

- Kyle Claassen, Rose-Hulman Institute of Technology
- Rick Laugesen, University of Illinois Urbana-Champaign
- Shane Leib, University of Notre Dame

Moderator:

- Justin Lambright, Anderson University

Mentoring Students for Non-Academic Careers
Most university and college faculty have spent their entire educational and professional journeys within academia. This panel seeks to provide insight and recommendations for faculty in mentoring students who are interested in pursuing careers in Business, Industry, and Government, and others outside our own personal experiences.

4. Contributed Talks

Presenter: Timur Akhunov, Wabash College
MSC 2020: 35H10, 35H20, 35S05, 35G05, 35B65
Derivative gain and singularities for degenerate Laplace equation
Many natural phenomena, from oil exploration to weather prediction to finance, are modeled with differential equations (DE). The Laplace equation plays a unifying role in the world of DE. Its solutions, famously, do not have singularities, which for a related heat equation means that spontaneous boiling of the water does not happen. Singularities are fascinating. You can win $1M if you settle the question of singularities for fluid motion. I have been studying Laplace-type equations that can have singularities for more than a decade. What is different about them?

Presenter: Mark Curiel, University of Hawaiʻi at Mānoa graduate student
Mixed volume of networks with binomial steady-states
Mass-action kinetics on a chemical reaction network gives rise to a polynomial dynamical system. The number of complex solutions to this system is called the steady-state degree — it is a measure of the algebraic complexity of solving the system. While the steady-state degree is difficult to compute in general, the mixed volume of the system can provide a decent upper bound. We exploit the geometry of partitionable binomial networks to give a method for computing the mixed volume via a matrix determinant.

Presenter: David Housman, Goshen College
Preferences in game theory
Game theory is the mathematical modeling of scenarios involving two or more decision makers, and it has been used successfully in economics, political science, biology, and other disciplines. Nonetheless, some game theory models have also been criticised for failing to accurately predict actual real-world behavior. The goal of this talk is to suggest that game theory models and solutions may not be at fault. Instead failures may be due to unwarranted assumptions about decision maker preferences over possible outcomes. This will be illustrated with a simple scenario in which the outcomes involve audience members receiving money, the amount depending on their choices.

Presenter: Daniel Maxin, Valparaiso University
Asymptotically Differential Equations
Sometimes, when solving a differential equation of the form $y'(t) = f(t, y(t))$, we are only interested in the limit of its solutions. Can we simplify this problem by taking the limit of $f(t, y(t))$ first? Often this results in a simpler autonomous equation of the form $y'(t) = g(y(t))$. Is it true that the limit of the solution of the limiting differential equation is the same as the limit of the solution of the original one? Surprisingly the answer is, sometimes, negative despite the fact that a large number of research papers actually take the affirmative answer for granted. I will discuss this aspect with several examples, some of them inspired from mathematical biology models.

Presenter: Tamsyn Morrill, Trine University
The Knave's Cosmological Theorem
The Look-Say sequence is a classic example of a recursive sequence. Its terms are verbalized descriptions of their predecessors — initialized at 1 — 11, 21, 1211, and so on. Conway demonstrated that the asymptotic growth rate of this sequence is an algebraic integer, the real root of a degree 71 polynomial. The general strategy is to recast the growth problem in terms linear algebra via his so-called Cosmological Theorem.

We present a variation of this problem. A knave (of Smullyan's famed doorkeeper puzzle) now controls the recursion. By developing a higher-order alphabet to the problem, we remake the Cosmological Theorem in the knave's image.

Presenter: Ikenna Nometa, University of Hawai‘i at Mānoa graduate student
Joint work with: Aida Maraj, University of Michigan, Jane Coons, University of Oxford, and Shelby Cox, University of Michigan
Maximum likelihood degrees of Brownian motion tree models
We give a survey of the progress made in the computation of the dual maximum likelihood (ML) degree of Brownian motion tree models. The major part of the work will focus on efforts to compute the ML degree of Brownian motion tree models and progress made on finding this number for star trees.

5. Contributed Posters

Presenter: Zhenisbek "J." Assylbekov, Purdue University Fort Wayne
MSC 2020: 68T05
Long-tail theory under Gaussian mixtures
We suggest a simple Gaussian mixture model for data generation that complies with Feldman's long tail theory (2020). We demonstrate that a linear classifier cannot decrease the generalization error below a certain level in the proposed model, whereas a nonlinear classifier with a memorization capacity can. This confirms that for long-tailed distributions, rare training examples must be considered for optimal generalization to new data. Finally, we show that the performance gap between linear and nonlinear models can be lessened as the tail becomes shorter in the subpopulation frequency distribution, as confirmed by experiments on synthetic and real data.

Presenter: Jasmine Carpena, University of Hawai'i at Mānoa undergraduate student
Faculty Advisor: Elizabeth Gross, University of Hawai'i at Mānoa
Network sampling
In this research project, we study graphical models, with a particular emphasis on preserving the maximum likelihood threshold. The background of our investigation lies in the crucial role of graphical models, which are used for representing complex relationships among variables. For example, in biology they are routinely used to infer gene regulatory networks from gene expression data and to infer co-occurrence networks from microbiome data. A pivotal aspect of these models, which we intend to explore, is that the existence of a maximum likelihood estimate (MLE) is contingent upon having a sufficient number of data points relative to the number of variables in the model. Our primary hypothesis and objective revolve around the notion that by harnessing combinatorics and algebraic statistics, we can optimize model selection routines to accommodate relatively small datasets. These datasets, characterized by fewer observations than random variables, are frequently encountered in practical applications. We aspire to uncover strategies and techniques that enable us to adapt model selection algorithms to the constraints of small-sample scenarios in a way that maintains the validity of maximum likelihood estimation.

Presenter: Dennis G. Collins, University of Puerto Rico, Mayagüez (retired)
Drama triangle cycling as chemical oscillation

This poster follows up on a presentation "Coalition, puzzle solving, and drama triangle cycling" at the April 1, 2023 MAA (Math. Assoc. of America) Indiana Sectional Meeting at IU Kokomo, IN. To review: A struggle between coalitions, such as ancient Greek city states with for example Delian League of Athens versus Sparta, can be studied by game theory; to avoid such struggles monotheism can suppose there is only one picture or map, which can be gradually filled in without conflict, as filling in different features of a puzzle; finally to be more realistic, it may be supposed that the one god tries to make everyone happy by cycling between coalitions on the way to resolution. Chemical systems, such as the Belousov-Zhabotinsky, Briggs-Rauscher, and Bray-Liebhafsky can exhibit such oscillations. Here SIL (Status-in-Life) is replaced by chemical concentration and rate equations solved. New cases include Reign of Terror in the French Revolution, the Nguyens in Viet Nam, and recently Putin-Prigozhin-Belarus dictator. More complicated cases generally exhibit chaos.

Presenter: Natalia Luna, Saint Mary's College undergraduate student
Joint work with: Molly Mullett, Saint Mary's College undergraduate student
Faculty Advisor: Pushpi Paranamana, Saint Mary's College
MSC 2020: 94A99
Belief evolution over time in social networks

Evolution of beliefs of a society is a result of interactions between people in the society over generations. We analyze the long term dynamics of belief evolution by combining people's prior beliefs, social dynamic network structures, and the confusion that occurs between beliefs. The main contribution of this work is threefold. First, we explore the belief evolution using existing network models such as scale free networks and small world networks to create social network structures and belief confusion structures. Second, we model the belief evolution with homophily based models using different statistical distances. We compare the individual and societal belief distributions and trends obtained from different models. Third, we explore the evolution of religious affiliations in different countries; both large and small in size, located in different continents. We use a homophily based model to fit religious affiliation data to model the dynamics of religious beliefs of Australia, Canada, and Ireland over time.

Using existing network models, we observe that the society evolves to a homogeneous belief system across all of the different models. However, the formation of heterogeneous belief systems such as social groups that share the same beliefs and isolated individuals can be observed using homophily based models. Moreover we can see how those formations change based on different distance measures. Finally, the successful implementation of the real world data justifies the theoretical formulation of the model and this allows us to interpret the social dynamics over time and the future implications.

Presenter: Dallas Pinkerton, Ball State University graduate student
Faculty Advisor: Dan Rutherford, Ball State University
A synopsis of (framed) holonomic knots
This poster will serve to familiarize the topic of holonomic knots through a brief study of Ekholm and Wolff's paper titled "Framed holonomic knots," as well as Vassiliev's "Holonomic links and Smale principles for multisingularities." Specifically, the poster will give an introduction to what a holonomic knot is, through rigorous definitions as well as examples. Additionally, we explore the properties framed holonomic knots have, as well as their relation to knots in general. Finally, proofs will be given for the statements of fact by the authors which go without proof in their respective papers.