Making beautiful music in the mathematics classroom

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INMAA Online

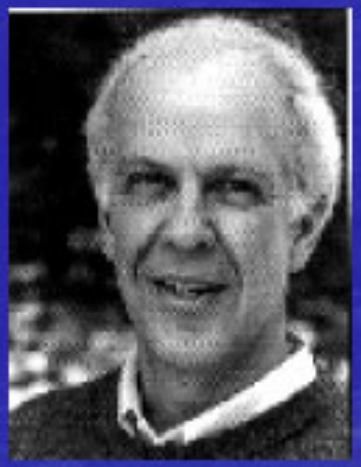
Based on an Alder Award Presentation
at Mathfest 2005

A Surprising Email

Identity and Integrity

Here is a secret hidden in plain sight: *good teaching cannot be reduced to technique; good teaching comes from the identity and integrity of the teacher.*

– Parker Palmer, *The Heart of a Teacher*

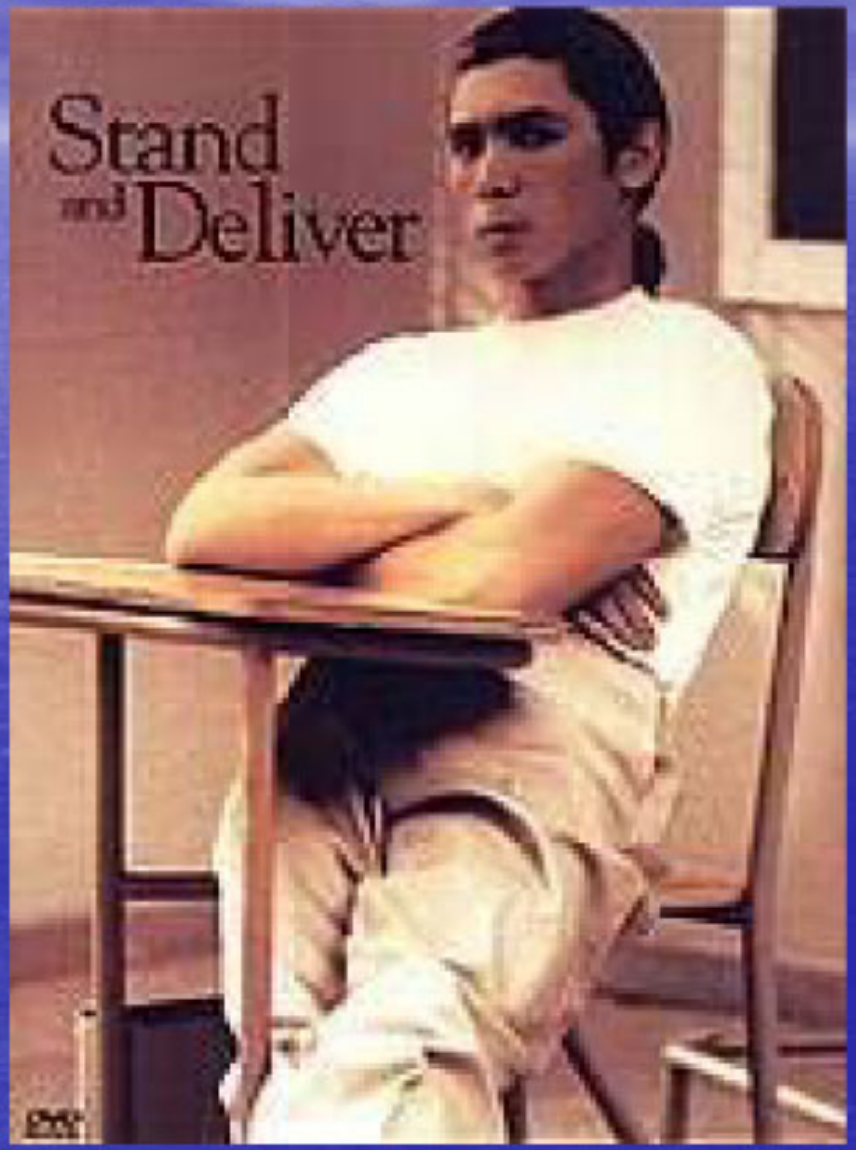


My Identity

- I'm a mathematician:
  Taylor University Professor and
  University of Michigan Ph.D.
- My father was a high school math teacher
- I'm a musician:
  Singer, choir director, faculty quartet
- I come from a family of amateur musicians (I have a brother with a graduate degree in music)

Les Choristes

Stand and Deliver



My Integrity

- I teach at a Christian college where foundational discussions are encouraged and beauty is accepted as a guide to truth and as a motivation to study.
- Finally, brothers, whatever is true, whatever is noble, whatever is right, whatever is pure, whatever is lovely, whatever is admirable—if anything is excellent or praiseworthy—think about such things.—Philippians 4:8

An Image Problem

Last time, I asked: "What does mathematics mean to you?" And some people answered: "The manipulation of numbers, the manipulation of structures." And if I had asked what music means to you, would you have answered: "The manipulation of notes?"—Serge Lang, *The Beauty of Doing Mathematics*

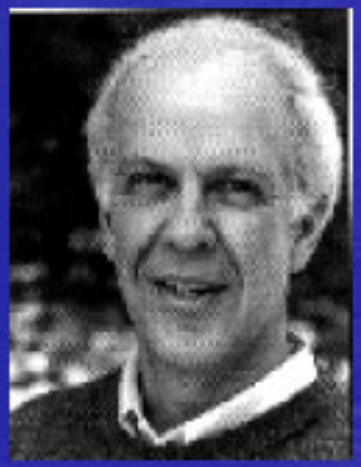
One Possible Solution

If we view beauty as central to mathematics we would be introducing ideas that, while they may have application or may build some tools, would certainly let students see a much bigger picture of mathematics at a much earlier stage in their mathematical development.—Sam Stueckle, *Beauty in Mathematics*

Passion

We became teachers for reasons of the heart, animated by a passion for some subject and for helping people learn.

– Parker Palmer, *The Heart of a Teacher*



Technique and Passion

Teaching mathematics and making music are similar in that they require both **good technique** and **passion** to be most effective.

Technique

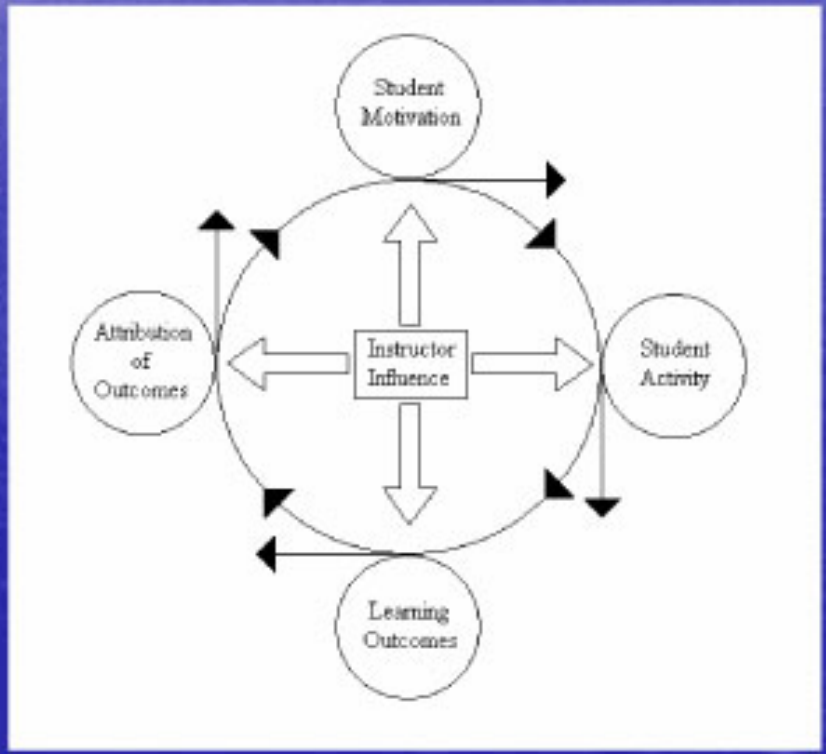
- Lecture
- Cooperative learning
- Appropriate use of technology
- Answering questions
- Etc.

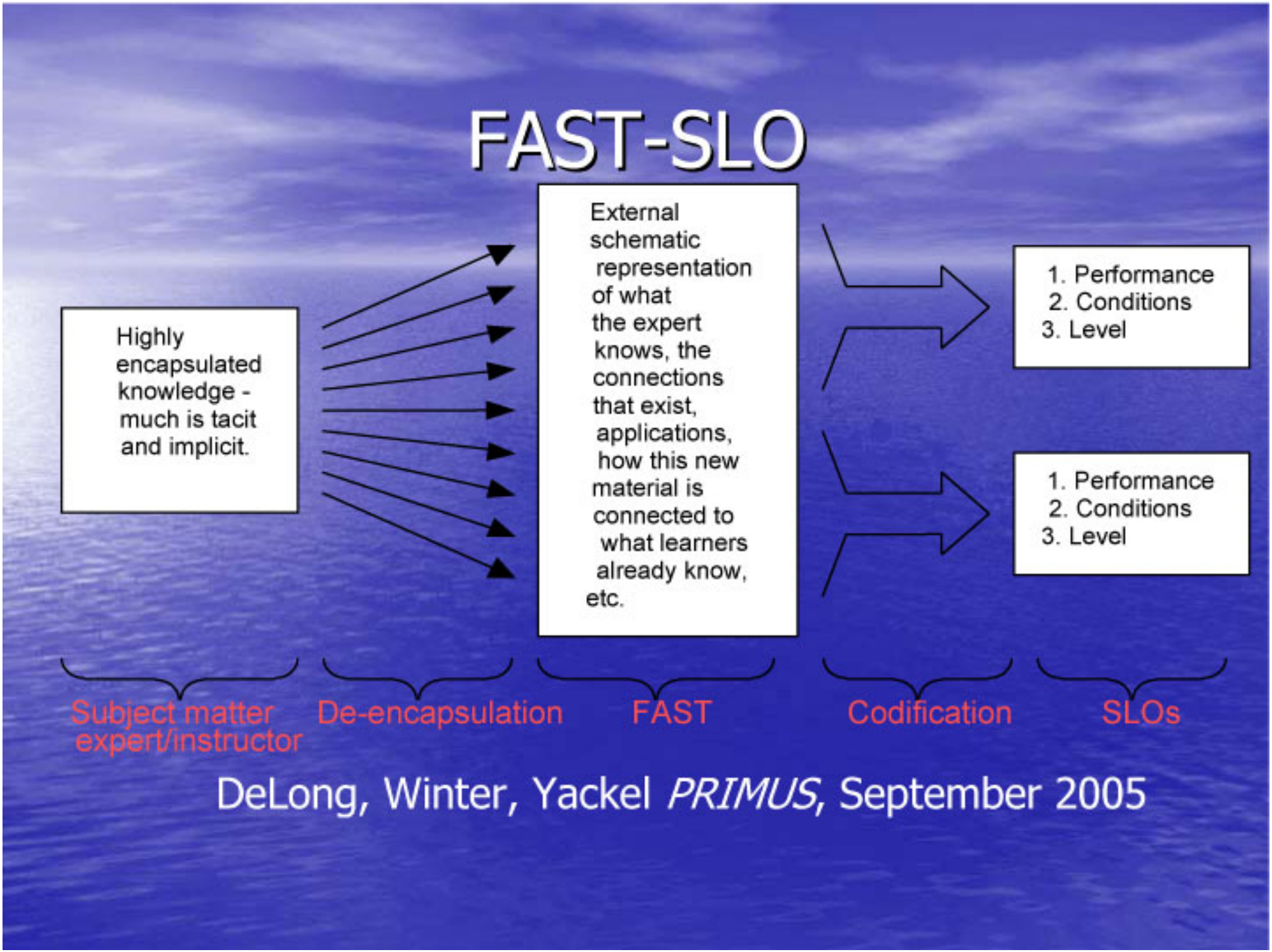
MALA

The MALA framework is a tool for focusing discussions about classroom management.

- Motivation
- Activity
- Learning outcomes
- Attribution

DeLong, Winter, Yackel
PRIMUS, June 2003




FAST-SLO
Highly encapsulated knowledge - much is tacit and implicit.
External schematic representation of what the expert knows, the connections that exist, applications, how this new material is connected to what learners already know, etc.
1. Performance
2. Conditions
3. Level
1. Performance
2. Conditions
3. Level
Subject matter expert/instructor
De-encapsulation
FAST
Codification
SLOs
DeLong, Winter, Yackel *PRIMUS*, September 2005

Beauty in Mathematics

Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show—Bertrand Russell, *The Study of Mathematics*

Beauty in Mathematics II

The mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas, like the colours or the words must fit together in a harmonious way.—G. H. Hardy, *A Mathematician's Apology*

Beauty in Mathematics III

It is more important to have beauty in one's equations than to have them fit experiment.—Paul Dirac, *Scientific American*

Beauty in Math IV

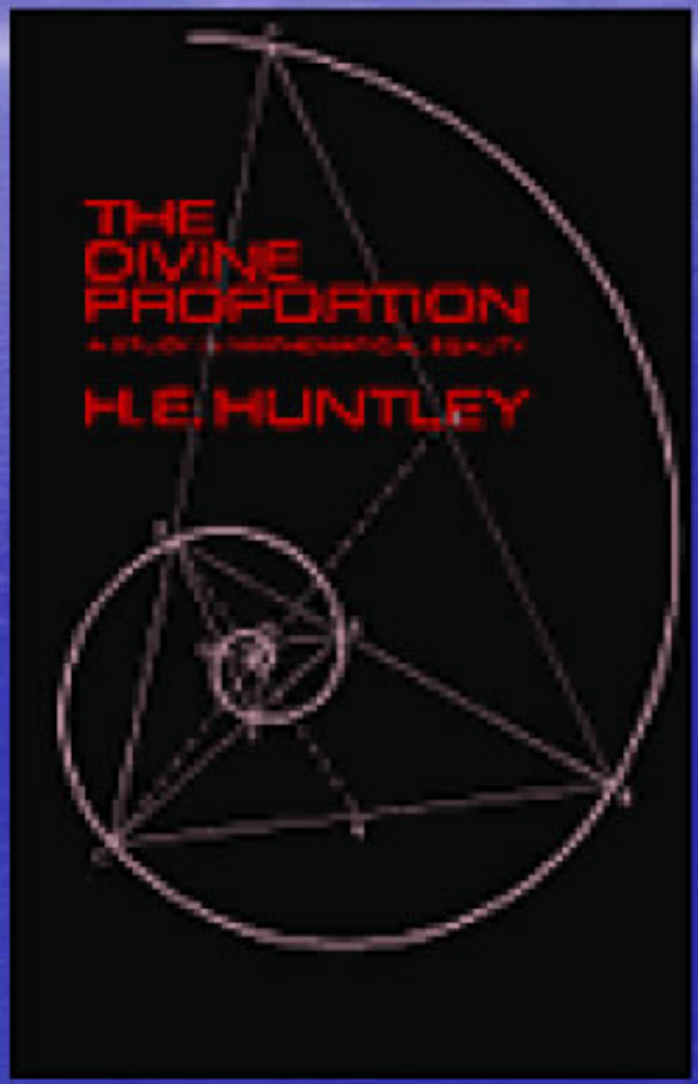
You get these feelings of beauty …

it's more about the whole story that was just told about a certain area of truth. You get this feeling of enlightenment…. Something that was sort of unclear in your mind before suddenly falls into place. You feel like you've learned a truth. You've gotten closer to understanding the world.—Manjul Bhargava, *Morning Edition*

Definitions

- Beauty is that quality or combination of qualities which affords keen pleasure to the senses, especially that of sight, or which charms the intellectual or moral faculties.—*Shorter Oxford English Dictionary*
- Beauty is that which pleases in mere contemplation.—Thomas Aquinas

Attributes of Beauty



Surprise at the unexpected

- Mathematics:
- Music: McClure's *Kyrie*

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \approx \frac{\pi^2}{6}$$

$$1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + - \cdots =$$

$$1 - \left(\frac{1}{\pi^2} + \frac{1}{4\pi^2} + \cdots \right) x^2 + (\cdots) x^4 + \cdots$$

Perception of relationships

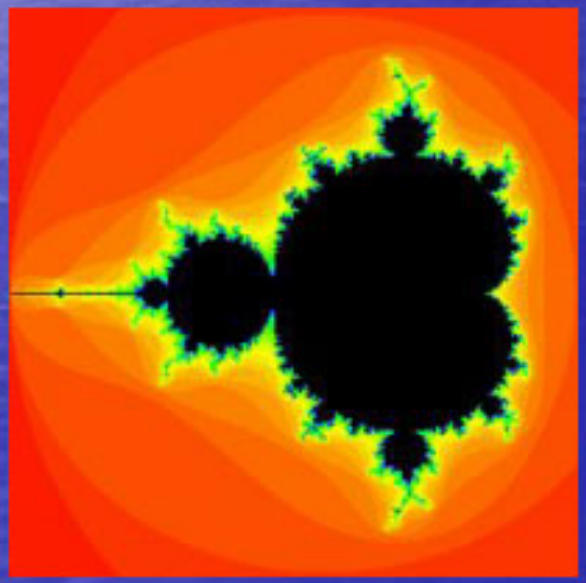
$$e^{i\pi} + 1 = 0$$

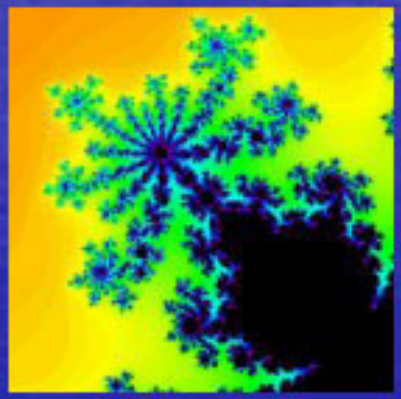
- Mathematics:
- Music: Hampton's *Praise His Holy Name*

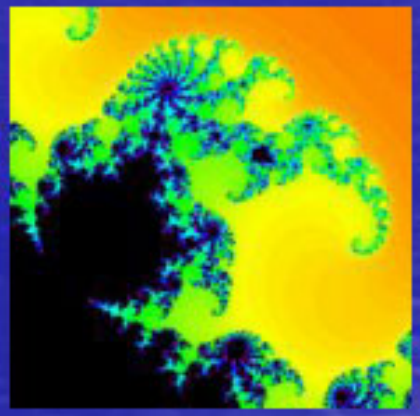
$$e^{ix} = \cos x + i \sin x$$

Organized complexity

- Mathematics: Mandelbrot Set
- Music: *He, Watching Over Israel*, Mendelssohn

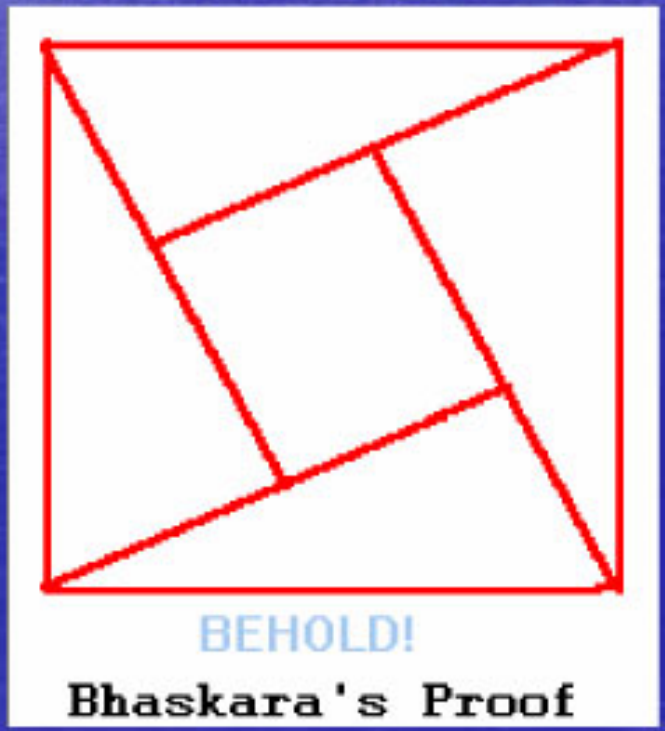






Brevity/Simplicity

- Mathematics: Bhaskara's Proof
- Music: Thomas's African Noel



Elegance

- Mathematics: Euclid's proof of the infinitude of primes
- Music: Mozart's *Ave Verum Corpus*

$$N = p_1 p_2 \cdots p_n + 1$$

Awe at the presence of the infinite

- Mathematics: Cantor's proof of the non-denumerability of the continuum
- Music: Handel's *Messiah*

$$b = .b_1 b_2 b_3 b_4 \ldots$$

$$b_n \neq 0,9, a_{nn}$$

But Can It Be Taught?

It's like asking why Beethoven's Ninth Symphony is beautiful. If you don't see why, someone can't tell you. I *know* numbers are beautiful. If they aren't beautiful, nothing is.—Paul Erdös

But Can It Be Taught? II

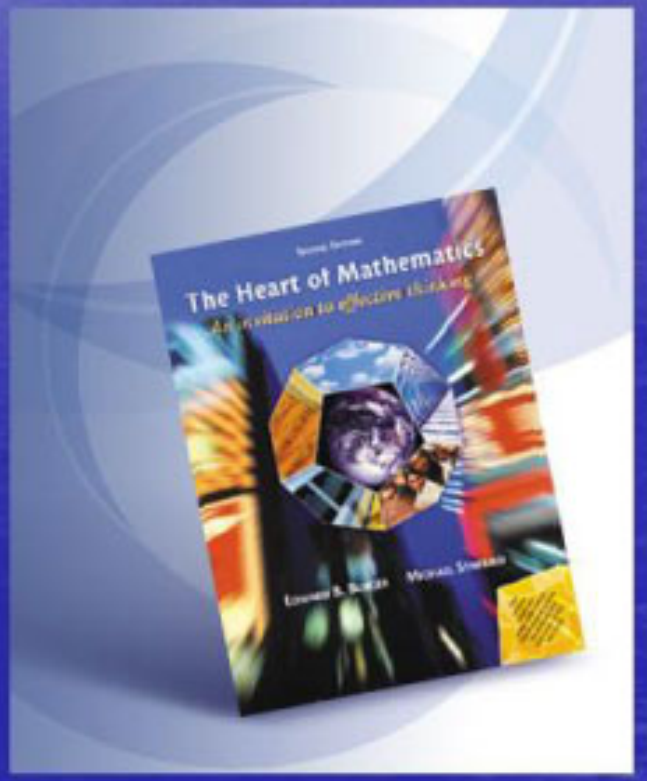
The mathematical esthetic sense is not distributed uniformly among the population, and there is a large number of people who find mathematics not only not beautiful, but repulsive.... There is nothing to be done about those who cannot appreciate mathematics.—Underwood Dudley, *Indiana Mathematics Teacher*

But Can It Be Taught? III

Music is passionate and very sensual. Mathematicians may see math that way, but try to explain that to anyone else and you sound like an idiot.—
Dan Naiman, *Johns Hopkins Magazine*

Investigations in Mathematics

- General education mathematics course for liberal arts majors
- **Course objective:** To help each student
  - uncover the beauty of mathematics and become fascinated by it,

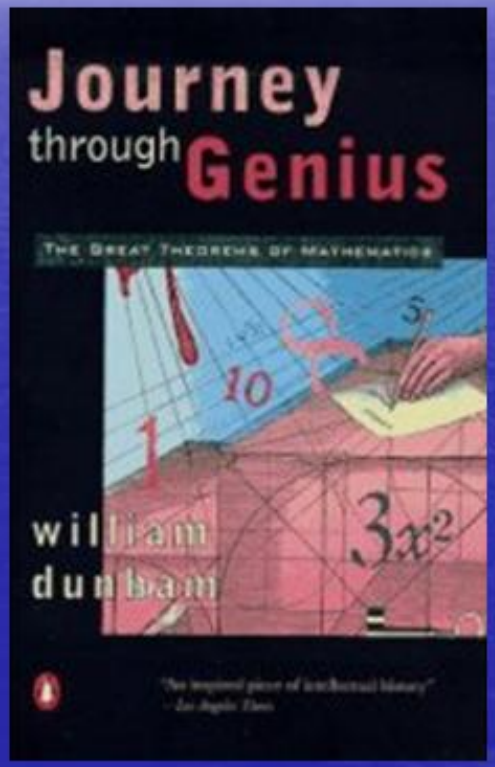


Investigations Feedback

- My attitude towards mathematics has improved as a result of this class.
  - 26 % Strongly Agree, 61 % Agree
- I enjoyed this course.
  - 26 % Strongly Agree, 61 % Agree

Problem Solving

- Required freshman course for all mathematics majors
- **Course objective:** To help each student
  - uncover the beauty of mathematics and become fascinated by it,

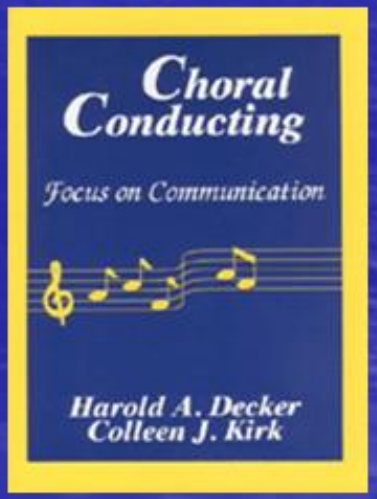


Problem Solving Feedback

- I have a better appreciation for the beauty of mathematics than I did before taking this course.
  - 18 % Strongly Agree, 73 % Agree

Lessons From Conducting

The successful conductor … assumes responsibilities as an interpreter of music, a teacher of skills, an organizer of experiences, a self-motivated learner, and a sensitive human being.—Decker and Kirk, *Choral Conducting: Focus on Communication*



Interpreter

- Knows the material extremely well
- Communication is unencumbered and clear (not confusing)
- Is thoroughly prepared
- Gives sections importance
- Makes choice of "repertory"

Teacher of skills

- Skillful themselves
- Understands student development
- Ably guide skill development
- Puts skills into context

Organizer of experiences

- Encourages growth and enjoyment
- Guides love of the discipline
- Encourages appreciation of the discipline as art
- Guides ability to read and interpret
- Encourages working together

Self-motivated learner

- Engages in exploration, research, learning
- Has a fascination with discovery

Sensitive human being

- Understands the diverse interests of the students
- Responds to the interests of the students
- Accepts responsibility for broadening the students' horizons
- Guides students in an ever-increasing awareness of their responsibility in learning

Making Beautiful Music

… mathematics should be taught as music is taught. Students should make mathematics *together* … not alone. Creative ideas should be stressed over the "right way to do it." … And finally, students should perform mathematics; they should *sing* mathematics and *dance* mathematics.—Jim Henle, *American Mathematical Monthly*