ABSTRACTS OF TALKS PRESENTED TO THE INDIANA SECTION OF THE MAA

1. Introduction

The Spring 2021 meeting of the Indiana Section of the Mathematical Association of America is being held virtually, hosted by the MAA, March 27. The abstracts appearing here are based on text electronically submitted by the presenters.

Contributed talks are listed in alphabetical order by presenter.

2. Invited Talks

Presenter: Satyan Devadoss, University of San Diego

Math Mysteries at Burning Man

See an image of *Unfolding Humanity* at `https://satyandevadoss.org`

Rising 12 feet tall with an 18-foot wingspan, a 2-ton unfolding dodecahedron comes to life at Burning Man, the world's most influential large-scale sculpture showcase. The artwork is illuminated by 16,000 LEDs with an interior large enough to hold 15 people and fully-lined with massive mirrors. This interactive sculpture alludes to a tantalizing open problem in mathematics on unfolding polyhedra, tracing its origins back 500 years to the Renaissance master Albrecht Dürer. We discuss the state-of-the-art for this geometric puzzle, consider solutions to some higher-dimensional unfolding analogues, and place this example in a larger quest to bring the edge of mathematical knowledge to the general public.

Presenter: James Sellers, University of Minnesota, Duluth; and Secretary of the MAA

Revisiting What Euler and the Bernoullis Knew About Convergent Infinite Series

All too often in first-year calculus classes, conversations about infinite series stop with discussions about convergence or divergence. Such interactions are, unfortunately, not often illuminating or intriguing. Interestingly enough, Jacob and Johann Bernoulli and Leonhard Euler (and their contemporaries in the early 18^{th} century) knew quite a bit about how to find the *exact* values of numerous families of convergent infinite series. In this talk, I will show two sets of *exact* results in this vein. The talk will be accessible to anyone interested in mathematics.

3. Indiana Project NExT Panel Discussion

Panelists:

- Justin Lambright, Anderson University
- Eric Reyes, Rose-Hulman Institute of Technology

Moderator:

- Chris Caruvana, Indiana University Kokomo

Reflections on Pandemic Teaching Innovations: A discussion of some lessons learned by us during these unprecedented times

During these unusual times, we have all had to adjust our teaching to fit the current situation. In this discussion, we'll hear from two of our colleagues about some of the changes they implemented, and we'll open discussion to anyone else who would like to share or reflect.

4. Student Activities Workshop

Presenters: Glen Van Brummelen, Trinity Western University, and Colin McKinney, Wabash College.

A Friendly Introduction to Spherical Trigonometry

In this workshop, we'll explore the exciting world of geometry and trigonometry on the sphere. Some things work the way you'd expect, but others are totally different from the Euclidean plane: the angles of a triangle, for example, always add up to more than 180°! In this system, we'll look at the foundations of the geometry and trigonometry, and look at the surprising statement of the spherical version of Pythagoras' theorem. We'll also look at Napier's *pentagramma mirificum* (amazing pentagram) and motivate the true origin of logarithms.

5. Contributed Talks

Presenters: Laura Batts and Megan Moran, Anderson University undergraduate students

Faculty Advisor: Courtney Taylor, Anderson University

Non-realizable root sequences for quintic polynomials

Rolle's theorem is a classical result typically studied in first-semester Calculus that can be used to locate the roots of a derivative of a polynomial between two consecutive roots of the polynomial. We investigate polynomials with distinct real roots, whose derivatives also have distinct real roots, none of which coincide. By differentiating several times, we produce a sequence of the roots of all nontrivial derivatives of a polynomial. For a polynomial of degree n this process produces $n(n+1)/2$ distinct roots. Although the ordering of the roots is constrained by Rolle's theorem, surprisingly not all root sequences allowed by Rolle's theorem exist. We investigate these non-realizable root sequences and establish elementary proofs for the non-realizability of particular root sequences.

Presenter: Dennis G. Collins, University of Puerto Rico, Mayagüez (retired)

Quaternion S_4 color coding — What is the square root of red?

There is recent interest in quaternion color coding, as a section of the recent annual AMS meeting shows. This talk follows up the speaker's Oct. 7, 2017 MAA Indiana Section talk at Manchester University on "AERO Cube S_4 group theory blocks," whereby it is shown the (quaternion) plane orientations can be extended to a color coding via quaternions with bank squared = I =red, yaw squared = J = green, and attitude squared = K = blue. As a consequence, for example, there are two square roots of red and the 24 group members cover most colors excepting cyan, magenta, and yellow, although the method could be based on these colors.

Presenters: Jessica Doctor, Timothy Hodges, Alexander McFarland, Taylor University undergraduate students

Faculty Advisor: Derek Thompson, Taylor University

Uniformly convergent iteration of quadratic functions of the unit disk

A theorem in complex analysis, the Denjoy-Wolff theorem, guarantees that nearly all analytic functions that map the open unit disk into itself, when iterated, will converge to a single point. Moreover, they do so uniformly on compact subsets of the disk. However, for some analytic functions, this convergence is uniform on the entire open disk, rather than just on compact subsets. We show that a family of quadratic functions has this property and explore the implications of this property.

Presenter: Zachary Gates, Wabash College

Joint work with: Robert Kelvey, The College of Wooster.

MSC 2020: 05, 20

Relator games on groups

We define two impartial games, the *Relator Achievement Game* `REL` and the *Relator Avoidance Game* `RAV`. Given a finite group G and generating set S, both games begin with the empty word. Two players form a word in S by alternately appending an element from $S \cup S^{-1}$ at each turn. The first player to form a word equivalent in G to a previous word wins the game `REL` but loses the game `RAV`. In this talk we will mostly think of `REL` and `RAV` as *make a cycle* and *avoid a cycle* games on the Cayley graph $\Gamma(G, S)$ as we discuss winning strategies for several families of finite groups including dihedral, dicyclic, and products of cyclic groups.

Presenter: Stacy Hoehn, Franklin College

Illuminating the math in the movies: A course on the mathematics of computer animation

When watching computer animated films, most people probably don't think about mathematics, but in reality, it would not be possible to bring scenes in these types of films to life without algebra, trigonometry, geometry, and other areas of mathematics. For example, animators can build complicated realistic shapes from basic geometric building blocks, algebra and trigonometry can be used to manipulate the size and position of images on the screen, and ideas from calculus can be used to smoothly transition from one image to another when animating a scene. In this talk, I will describe a course taught at Franklin College in which students explored these ideas and then created their own short animated movies using their newly acquired skills.

Presenter: David Housman, Goshen College

MSC 2020: 91B32

Congressional apportionment

The United States Constitution mandates a census every ten years to determine the number of seats each of the fifty states will hold in the House of Representatives. This might seem to be an elementary exercise in arithmetic, requiring merely a simple rounding of fractions. This has not been the case, however. Apportionment methods have been debated frequently and at length. Alexander Hamilton, Thomas Jefferson, Daniel Webster, and Edward Huntington are among the many who have recommended different methods. History and common sense suggest that any apportionment method should satisfy certain fairness properties. For example, (1) if a state deserves 6.3 seats, it should receive 6 or 7; (2) if the total number of seats available is increased, no state should lose a seat; (3) there should be no systematic bias between small and large states; and (4) if state A's population grows faster than state B's population, then it should not be the case that state A loses a seat while state B gains a seat. Unfortunately, no apportionment method can satisfy all fairness properties that have been suggested. Different apportionment methods can be characterized using different sets of fairness properties. The selection of an apportionment method can then be based upon arguments about what constitutes fairness rather than what affords the greatest political advantage.

Presenters: Hallie Kaiser, Katy O'Malley, Grace Weeks, Taylor University undergraduate students

Faculty Advisor: Derek Thompson, Taylor University

Normality properties of composition operators

We explore two main concepts in relation to truncated composition matrices: the conditions required for the binormal and commutative properties. Both of these topics are important in linear algebra due to their connection with diagonalization. We begin with the normal solution before moving onto the more complex binormal solutions. Then we cover conditions for the composition matrix to commute with a general matrix. Finally, we end with ongoing questions for future work.

Presenters: Chloe Makdad, Ben Rempfer, and DeJuan Winters, Butler University undergraduate students

Faculty Advisor: Scott Kaschner, Butler University

Uniform convergence of iterates on the unit ball in higher dimensional domains

For nontrivial self maps of the complex unit disk, the Denjoy-Wolff theorem guarantees that sequences of iterates of such a map converge uniformly on compact subsets of the disk to a unique fixed point. Recently, Cowen, Ko, Thompson, and Tian developed conditions on these maps for when this convergence is uniform on the whole unit disk. We explore the analogous problem in a variety of several variables contexts: $\mathbb{C}^n$ with $n > 1$, the bicomplex numbers, and the quaternions.

Presenter: Christina Pospisil, USA undergraduate student

MSC 2020: 47

Generalization theory of linear algebra III

This talk continues the presentations Generalization Theory of Linear Algebra I+II from JMM 2019 Conference and JMM 2020 Conference [*and Indiana MAA Section Meetings* — ed.]. In the first part, an algorithm for multiplying and adding regardless of dimensions via an embedding and inverses for non-injective mappings in one dimension were presented. The second part presented inverses for non-injective mappings in multiple dimensions, inverses for non-surjective mappings in one and multiple dimensions, and introduced a general determinant theory. The third part is dedicated to a further generalization regarding tensors with first applications in physics. In future work, further operations and applications to physics and other natural sciences will be explored.

Presenter: Isaac Reiter, Kutztown University of Pennsylvania undergraduate student

Faculty Advisor: Ju Zhou, Kutztown University of Pennsylvania

What you have leftover is perfect: Enumerating the PSR divisors

There are certain positive integers that, when divided into any perfect square, always yield a remainder that is a perfect square. For example, when 12 is divided into a perfect square, the remainder is 0, 1, 4, or 9. We will refer to these numbers as PSR divisors. In this talk, the presenter will prove that there are only eight PSR divisors among the positive integers.

Presenter: Melvin Royer, Indiana Wesleyan University

Supertask paradoxes of the infinite

Paradoxes of the infinite have fascinated, frustrated, and amused mathematicians and philosophers since the times of the ancient Greeks. This expository talk will examine some of my favorite paradoxes such as the Pi Machine, Achilles and the Gods, and Ross' Urn. Technology permitting, the audience will be able to vote their interpretation and understanding of these "supertask" paradoxes. I will attempt to explain (spoiler alert: unsuccessfully) how these paradoxes can or cannot be resolved according to various philosophers of mathematics.

Presenter: Alessandro M. Selvitella, Purdue University Fort Wayne

The ubiquity of Simpson's paradox

Simpson's Paradox is the phenomenon that appears in some datasets, where subgroups with a common trend (say, all negative trend) show the reverse trend when they are aggregated (say, positive trend). Even if this issue has an elementary mathematical explanation, it has a deep statistical significance. In this talk, we will discuss basic examples in arithmetic, geometry, linear algebra, statistics, game theory, gender bias in university admission, and election polls, where we describe the appearance or absence of Simpson's Paradox. If time permits, we will present our results concerning the occurrence of Simpson's Paradox in quantum mechanics.

Presenter: Laura Shevlin, Hawkes Learning

MSC 2020: 97

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