ABSTRACTS OF TALKS PRESENTED TO THE INDIANA SECTION OF THE MAA

1. Introduction

The Spring 2016 meeting of the Indiana Section of the Mathematical Association of America is at Franklin College, March 18–19. The abstracts appearing here are based on text electronically submitted by the presenters. Contributed talks are listed in alphabetical order by presenter.

2. Invited Talks

Presenter: Edray Goins, Purdue University West Lafayette

Indiana Pols Forced to Eat Humble Pi — The Curious History of an Irrational Number

In 1897, Indiana physician Edwin J. Goodwin believed he had discovered a way to square the circle, and proposed a bill to Indiana Representative Taylor I. Record which would secure Indiana's claim to fame for his discovery. About the time the debate about the bill concluded, Purdue University professor C. A. Waldo serendipitously came across the claimed discovery, and pointed out its mathematical impossibility to the lawmakers. It had only been shown just 15 years before, by the German mathematician Ferdinand von Lindemann, that it was impossible to square the circle because π is a transcendental number. This fodder became ignominiously known as the "Indiana Pi Bill" as Goodwin's result would force $\pi = 3.2$.

In this talk, we review this humorous history of the irrationality of π. We introduce a method to compute its digits, present Lindemann's proof of its irrationality (following a simplification by Miklós Laczkovich), discuss the relationship with the Hermite-Lindemann-Weierstrass theorem, and explain how Edwin J. Goodwin came to his erroneous conclusion in the first place.

Presenter: Colin McKinney, Wabash College

Hoosiers, The MAA, and the Indiana Section

This talk will chronicle the role some Hoosiers played in the founding of the MAA. It will also look at the founding of the Indiana Section of the MAA, only the fifth section to form. Some biographical information will also be presented about early members of the Section or the Association.

Presenter: Jeffrey Oaks, University of Indianapolis

How to Think Like a Medieval Algebraist

So you think a polynomial is just a polynomial? You think that the objects of mathematics are universal abstractions that we all apprehend directly (sometimes after many dreadful hours of studying)? Through a look into the language and structure of the algebraic solutions to problems in premodern algebra (before ca. 1600) I show that their conceptions of the powers of the unknown, polynomials, and equations differ from ours, and that the different premodern notations reflect these conceptions.

Presenter: Jason Rosenhouse, James Madison University, Virginia

The Monty Hall Problem, Reconsidered

The Monty Hall problem is a classic brainteaser in probability. In its canonical form, it asks you to imagine that you are a game show contestant confronted with three doors. Behind one of the doors is a car, behind the other two are goats. Monty Hall, the host of the show, asks you to choose a door but not open it. After you make your choice, Monty then opens one of the remaining two doors, showing you that it contains a goat. He then gives you the options either of sticking with your original door, or switching to the one remaining option. You then win whatever is behind your final choice. What should you do, assuming you want to maximize your chances of winning the car? This problem routinely causes controversy, since the intuitively obvious answer turns out to be wrong. We shall discuss the mathematics underlying the problem, explain how to think clearly about problems in conditional probability, and discuss a series of increasingly complex variations on the basic scheme. The talk assumes very little mathematics and will be readily accessible to undergraduates.

3. Indiana Project NExT Panel Discussion

Panelists:

- Mindy Capaldi, Valparaiso University
- Sylvia Carlisle, Rose-Hulman Institute of Technology
- William Lindsey, IU-Kokomo

Moderator: Josh Holden, Rose-Hulman Institute of Technology

Alternative Assessment Techniques

In this panel, we will discuss methods to assess student learning that go beyond traditional exams. The panelists will first each give an overview of alternative assessment techniques they have tried in their classrooms, ranging from pre-class assignments and specifications grading to oral exams and collaborative testing. Then there will be time for Q&A from the audience.

4. Student Activities Workshop

Presenters: Karl Schmitt, Valparaiso University, and Martha Byrne, Earlham College

Math + Gaming = Mathing

We know you like math. Do you also like playing tabletop games? Ever thought about how much math goes into games? We're talking more than just the probability in dice rolling. You can find symmetry, sets, finite projective planes, axiomatic systems, and more. In this workshop we'll take a brief look at how mathematical concepts appear in or inspire games. After that you'll get the opportunity, guidance, and materials to start designing a new game based on mathematical principles with a team of like-minded individuals. We'll provide mathematical subjects for structure and example games for inspiration. You'll bring your creativity and love of mathematics to make playing with math as fun for everyone as it already is for us.

5. Graduate Student Workshop

Presenters: Amanda Harsy, Lewis University, and Mark Panaggio, Rose-Hulman

Applications and pre-campus interviews

Are you ready for the job market? This workshop will offer information and advice about the early stages of an academic job search with a focus on assembling your application materials and preparing for pre-campus interviews. It will include helpful tips, guided discussion, and mock interviews and writing prompts.

Topics covered in this session will include:

- The timeline of applying for jobs and where to find job postings,
- Advice for writing your CV, cover letter, teaching statement, research statement, and other application materials,
- How to obtain letters of recommendation — whom to ask and when,
- How to stay organized and manage deadlines,
- Guidelines for pre-campus interviews including: phone, Skype, and conference (JMM) interviews,
- Typical interview questions you should be prepared to answer, and
- Good questions to ask during your pre-campus interview.

6. Contributed Talks

Presenter: Lowell Beineke, Indiana University - Purdue University Fort Wayne

Joint work with: Jay Bagga, Ball State University

Parity matters in product-optimization when splitting integers

The following question arose in a problem we were working on in graph theory: Given integers $m, n \geq 2$, what is the maximum of the minimum of ac and bd if $a + b = m$ and $c + d = n$? The problem turns out to be much harder than it sounds, and depends on the parities of m and n. If both are even, the answer is the expected one, while if m is even and n odd with $m < n$, the answer is not always what one might expect, but has been found for all but finitely many values of n, given m. However, in the other two cases, when both m and n are odd, or when the smaller is odd and the larger even, the situation is more complicated, and not many exact results are known. We will present some of the results and the techniques used in proving them as well as the connection with graph theory.

Presenter: David Benko, University of South Alabama, and visiting IPFW

The grandest of all grand slams

Have you ever wondered which one of the four tennis grand slam tournaments is the best? The Australian Open? The Roland Garros? The US Open? Wimbledon? They are played on different surfaces: grass, clay, acrylic hard court, synthetic hard court, and under different weather conditions: sun, wind, rain, snow. ☺ How can we pick a winner? We use game statistics and a unique method to find the answer.

Presenter: Martha Byrne, Earlham College

Using games to engage students in inquiry

This presenter will talk about her experience using games as a vehicle for engaging students in mathematical inquiry. She will discuss exploring the mathematical principles underlying existing games (SET, Six, Spot it!, and Blokus) and engaging deeply with mathematical principles (axiomatic geometry and modular arithmetic) to design new table top games. She led these explorations at a small liberal arts college during the 2014–2015 academic year.

Presenter: Chris Foss, Indiana Wesleyan University undergraduate student

Connecting Stirling number to differences of powers

When repeatedly taking the differences of the n^{th} power of consecutive integers, the process stabilizes after n steps. During this talk, we will explore this and other patterns that arise from this process. In particular, we will describe two generating functions that help determine the values occurring in the intermediate steps of the process. We will also introduce the famous Stirling numbers of the second kind that are an integral part of one of the generating functions.

Presenter: Luke Hayden, Indiana Wesleyan University undergraduate student

Faculty Advisor: Daniel Kiteck, Indiana Wesleyan University

A combinatoric analysis of Abalone

Abalone is a two person strategy board game which is relatively unstudied from a combinatorics perspective. While prior researchers have focused largely on computer algorithms and developing a more efficient AI, this research focuses on Abalone from a different angle. New formulas are introduced to determine the total number of game state possibilities based on various factors. In addition, the high degree of symmetry of the game board leads to patterns for the number of unique game state possibilities. Finally, this research covers an algorithm which when used ensures a victory in a four marble versus one marble match-up.

Presenter: William Higdon, job-seeking mathematician

On the numerical ranges of composition operators on $H^2(D)$ induced by mappings having the Denjoy-Wolff point on the boundary

MSC 2010: 47A12

The *numerical range* of an operator T on a Hilbert space H, denoted $W(T)$, is the set,

$$W(T) = \{ < Tx, x > \ | \ x \in H \text{ and } \|x\| = 1\}.$$

Elementary properties of $W(T)$ include that it is a bounded subset of $\mathbb{C}$, it is convex, it contains the eigenvalues of T, and, more generally, its closure includes the spectrum of T. The work presented here answers the question posed by Professors Paul S. Bourdon and Joel H. Shapiro in their paper (*When is zero in the numerical range of a composition operator?*, Journal of Integral Equations and Operator Theory **44** (2002), 410–441):

"Suppose the symbol ϕ of the composition operator C_ϕ on $H^2(D)$ is univalent, not linear fractional, and is of parabolic nonautomorphism type. Is $0 \in W(C_\phi)$?"

The answer has been, in general, unknown. One property of such a mapping is that it has derivative equal to 1 at its Denjoy-Wolff point (boundary fixed point). Using elementary properties of C_ϕ-invariant subspaces, the answer to the question above can be demonstrated (in even simpler terms than those with which it is stated). Together with the work of Bourdon and Shapiro, this provides a complete description of when 0 belongs to the numerical range of a composition operator C_ϕ on $H^2(D)$.

Presenter: Ryan Johnson, Grace College

Using Gauss sums to distinguish certain algebraic structures

MSC 2010: 18D10

Fusion categories are complicated algebraic structures found in diverse branches of mathematics and physics. Quantum computing is one specific example that will be mentioned in this talk. Classification of fusion categories is a main driving question for mathematicians in this area. A natural method for classifying objects in mathematics is via numerical invariants. We will consider a subclass of fusion categories and a sequence of invariants of these structures. As it turns out, the invariants can be written as Gauss sums. Gauss sums have a rich, well-studied history, and we can also use Gauss sums to create interesting pictures. We use Gauss sums to prove that our invariants distinguish between different fusion categories in our subclass.

Presenter: Edna Jones, prospective graduate student
Joint work with: John Ryan, New York University
Faculty Advisor: Peter Johnson, Auburn University
Theoretical friends of finite proximity
MSC 2010: 11A25

For a positive integer n, the abundancy index of n is $I(n) = \frac{\sigma(n)}{n}$, where $\sigma(n)$ is the sum of the positive divisors of n. Two distinct positive integers are friends if they have the same abundancy index. A theoretical friend of proximity t of a positive integer m is a sequence $s = \{s_k\}_{k=1}^{\infty}$ of positive integers such that $\lim_{k\to\infty} I(s_k) = I(m)$; $|P_s| = t$, where $P_s = \{p : p \text{ is a positive prime and } p \text{ divides } s_k \text{ for some } k\}$; and $s_k \neq m$ for all k. We say that s is a theoretical friend of finite proximity (TFOFP) of a positive integer m if and only if s is a theoretical friend of m of proximity t for some nonnegative integer t.

In 2008, Jeffrey Ward posed the question: Does every positive integer have a TFOFP? Using some properties of the abundancy index, we show that the answer to this question is no. For example, 1 does not have a TFOFP. We show that no odd prime has a TFOFP and certain integers have a TFOFP if and only if they have a friend.

This work was done at the Research Experience for Undergraduates in Algebra and Discrete Mathematics at Auburn University.

Presenter: Michael Karls, Ball State University
Verifying the hanging chain model
MSC 2010: 35L, 35Q

The wave equation with variable tension is a classic partial differential equation that can be used to describe the horizontal displacements of a vertical hanging chain with one end fixed and the other end free to move. Using a web camera and Tracker software to record displacement data from a vibrating hanging chain, we verify a modified version of the wave equation with variable tension that accounts for damping.

Presenter: Daniel Kiteck, Indiana Wesleyan University
Inquiry-Based Learning general education math class
MSC 2010: 97

In Fall 2015 I taught a general education math class where a majority of the material came from the Inquiry-Based Learning material *Discovering the Art of Mathematics* (`artofmathematics.org`). My themes were number, shape, and infinity. The majority of class times students worked in small groups being guided through worksheets where the goal was students discovering the mathematics for themselves as much as possible. In this presentation I will reflect on the experience.

Presenters: Tiffany Kolba, Valparaiso University, and Shannon Segin, Valparaiso University undergraduate student

Modeling games with Markov chains

Markov chains are stochastic (i.e. random) processes in which the future state depends only upon the present state and not the past. This talk will focus on group projects in a stochastic processes course where students modeled real-world processes, such as sporting events, business operations, and games, with Markov chains. In particular, the talk will highlight Markov chain models for the games of Blackjack and Yahtzee.

Presenter: Crystal Lorch, Ball State University

Top Ten List: Overhauling a liberal arts mathematics course

A comprehensive course overhaul is a daunting task. The approach used at Ball State University to redesign the core quantitative reasoning course which serves 3000 students annually is outlined. Flipping the classroom is a prominent feature of the redesign. Key considerations are presented for the benefit of anyone contemplating such an undertaking.

Presenter: John McSweeney, Rose-Hulman Institute of Technology

Analyzing crossword puzzles with graph theory

We can model the structure of a crossword puzzle with a graph where vertices represent words, and the presence of an edge between two vertices indicates that the two words cross. In this way, we can attempt to quantify the difficulty of a puzzle in terms of how the structure of the puzzle grid interacts with the inherent difficulty of the clues. The solution propagates across the grid in a way analogous to how a disease spreads across a population, and we can therefore use the language and mathematics of epidemics to describe the spread (or lack thereof!) of a solution.

Presenter: Kiah Wah Ong, Indiana University graduate student

Phase transitions and bifurcations

Phase transitions and bifurcations are of central importance in nonlinear sciences. Some typical examples include the solid/liquid/gas transitions, segregation of block copolymers in a polymer melt, the onset of Rayleigh-Bénard convection, etc. In all these instances, there is a specific physical parameter which crosses a threshold, and the system undergoes a reorganization to a new state which is considerably different. In this talk, center manifold reduction strategy is introduced to address dynamic transitions of dissipative systems.

Presenter: Ranjan Rohatgi, Indiana University graduate student
Flip-mastery learning in applied calculus
MSC 2010: 97

In the fall of 2014, over 700 students participated in an experiment testing the efficacy of the flip-mastery model in teaching calculus. Four instructors taught two classes each — the first a traditional lecture-style class and the second a flipped class. Graduate students created an online content-delivery and homework system giving the students a single website on which they could learn new material and answer homework problems. In this talk I will show how we combined a flipped classroom with mastery learning techniques and discuss some preliminary results from the two sections I taught.

Presenter: Derek Thompson, Taylor University
Invariant subspaces in infinite dimensions

This talk will give a broad introduction to operator theory and function spaces, by generalizing some concepts from Linear Algebra and Calculus II, perhaps motivating student research projects in function spaces. Students who have had both courses should be able to follow along.

Presenter: Caryn Willis, Indiana Wesleyan University undergraduate student
Faculty Advisors: Melvin Royer and Bob Mallison, Indiana Wesleyan University
An extension of the Hassell-Comins discrete time model for two competing species

This research generalizes a discrete time competing species population model developed by Hassell and Comins. In their paper, these authors characterize the location and local stability properties for the equilibria of this dynamical system in terms of the model parameters. We extend their model to a more general form and present computer simulations suggesting new behaviors, some of which we prove. We also discuss the meaning of our model parameters and give biologically intuitive explanations of some of these behaviors.

Presenter: Pengcheng Xiao, University of Evansville
A theoretical analysis for extended minimal model of HPA axis
MSC 2010: 92B05

The hypothalamic-pituitary-adrenal (HPA) axis plays an important role under stressed conditions in raising the concentration of the HPA axis hormones, which leads to energy directed to the organism. In our term extended minimal model, we consider our network of cortisol in HPA axis with the GR and we assume the negative feedback are from GR to CRH and ACTH. This paper concerns ODE modeling of the HPA axis using an analytical approach.

Presenter: Michael Xue, Vroom Laboratory for Advanced Computing
A constructive proof of Euler's Line Theorem using a computer algebra system

Euler's Line Theorem states that in every triangle, the intersection of the medians, the intersection of the heights, and the center of the circumscribed circle are on a straight line. This talk offers an algebraic and algorithmic proof with the aid of a Computer Algebra System's (CAS) symbolic computation capabilities. We will use `Omega`, a free online CAS Explorer in this presentation.

Presenter: Michael Xue, Vroom Laboratory for Advanced Computing

Prove inequalities by solving maximum/minimum problems using a computer algebra system

This presentation offers an alternative to traditional approaches to proving non-trivial inequalities, such as applying AM-GM, Cauchy, and Jensen's inequalities. This alternative approach, as demonstrated by various examples, establishes the validity of an inequality through solving a maximization/minimization problem by commonly practiced procedures in Calculus. Since the procedures are algorithmic, a Computer Algebra System (CAS) can carry out the computation efficiently.

Presenter: Young Hwan You, Indiana University East

Three presences (social, cognitive, and teaching presence) as a framework in online education

With the rapid development of internet and computer-related technologies, the education area has broadened to a cyber-world beyond the traditional classrooms. In this talk, I would like to introduce three presences (social, cognitive, and teaching presence) as a framework in online education. In particular, I will talk about how these three presences can be adopted in online classes.