ABSTRACTS OF TALKS PRESENTED TO THE INDIANA SECTION OF THE MAA

1. Introduction

The Fall 2021 meeting of the Indiana Section of the Mathematical Association of America is being held virtually, hosted by the MAA, September 25. The abstracts appearing here are based on text electronically submitted by the presenters.

Contributed talks are listed in alphabetical order by presenter.

2. Invited Talks

Presenter: James Oxley, Louisiana State University

Geometry, Greed, Games, and 'Roids

All who have taken a high school geometry class have seen the construction of a tangent to a circle using a compass and ruler. But, can it be done without using a compass? Now suppose you want to find the cheapest way to connect a set of towns by a rail system. Can you find the answer quickly? Finally, suppose Destroyer and Constructor are playing a game on a connected network, moving alternately. Each of Destroyer's moves obliterates a link, while each of Constructor's moves makes a link indestructible. When can Constructor prevent Destroyer from breaking the network into pieces? This talk will answer these three questions and will discuss a common mathematical framework underlying them.

Presenter: Alain Togbé, Purdue University Northwest

Current Trends in Diophantine Sets

A set of m distinct positive integers $\{a_1, \ldots, a_m\}$ is called a Diophantine m-tuple if $a_i a_j + 1$ is a perfect square. In general, let n be an integer, a set of m positive integers $\{a_1, \ldots, a_m\}$ is called a Diophantine m-tuple with the property $D(n)$ or a $D(n)$-m-tuple (or a P_n-set of size m), if $a_i a_j + n$ is a perfect square. Diophantus studied sets of positive rational numbers with the same property, particularly he found the set of four positive rational numbers $\left\{\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}\right\}$. But the first Diophantine quadruple was found by Fermat. That is the set $\{1, 3, 8, 120\}$. Moreover, Baker and Davenport proved that the set $\{1, 3, 8, 120\}$ cannot be extended to a Diophantine quintuple. The problem of the extendibility of Diophantine m-tuples is of a big interest. During this talk, we will give a very quick history of m-tuples and discuss of the conjectures and the recent progress to solve these conjectures.

3. Indiana Project NExT Panel Discussion

Panelists:

- Zachary Gates, Wabash College
- Nayeong Kong, Indiana University East.

Moderator:

- Justin Lambright, Anderson University

Alternative Assessments Techniques
Zachary Gates will discuss Standards-Based Grading, drawing from his own experiences using this in multiple courses and also sharing what he has learned from others about this method. Nayeong Kong will then share with us Mathematics Outcome Assessments via Canvas, focusing on the assessment methods used recently in an Introduction to Probability Theory course. She will share various ideas to develop the assessment methods both in the classroom and in online courses.

4. Contributed Talks

Presenters: Christopher Barua and Gabe Fragoso, Valparaiso University undergraduate students
Determining the winner in a graph theory game
Our research focused on investigating who has the winning strategy in an original game described by Francis Su in his book, *Mathematics for Human Flourishing*. This game is played on a simple planar graph consisting of vertices and edges connecting them, and consists of two players taking turns directing edges, taking into account certain restrictions, to try and complete a cycle cell, where a cycle cell takes the shape of a polygon. We examined game boards where the winning strategy was previously unknown. Starting with a pentagon and a heptagon glued by two sides, we worked to solve multiple classes of graphs involving stacked polygons. We also explored variations of the game where cycles, as defined in graph theory, are used in place of cycle cells, which opens the game up to non-planar graphs, such as complete graphs and gives the game a graph theory twist on top of topology.

Presenter: Dennis G. Collins, University of Puerto Rico, Mayagüez (retired)
Consciousness toolkit
This talk covers the extent to which the 24 element group S_4 and its extension to the 48 element binary octahedron group (BOG) can be considered a toolkit for consciousness in the brain. It goes beyond the Author's Indiana Section MAA talk March 27, 2021 on "Quaternion S_4 color coding" to present quaternion BOG color coding and attempts to calculate the symmetry of the BOG group according to the Collins 2011 patent on measuring the symmetry of a data set. Problems with music theory are discussed, as well as the boundary between quantum and macro events. These questions are related to work by Louis Kauffman and others.

Presenter: José Contreras, Ball State University

The joy of investigating geometric converse problems with GeoGebra

In this presentation, I illustrate how my students and I use GeoGebra to discover the solutions to geometric converse problems. In particular, we use GeoGebra to gain insight into the solution to the following three problems.

(1) Let $ABCD$ be a quadrilateral with medial quadrilateral $EFGH$. If $EFGH$ is a rectangle, what type of quadrilateral is $ABCD$?
(2) Let E, F, G, and H be the midpoints of the consecutive sides of a quadrilateral $ABCD$. If $EFGH$ is a rhombus, characterize quadrilateral $ABCD$.
(3) E, F, G, and H are the midpoints of the consecutive sides of a quadrilateral $ABCD$. Name quadrilateral $ABCD$ when $EFGH$ is a square.

Presenters: Gabe Cowley and Kihyun Kim, Wabash College undergraduate students

Faculty Advisor: Katie Ansaldi, Wabash College

Rainbow numbers of $\mathbb{Z}_n$ for $x - y = z^2$

An exact $r-$*coloring* of $\mathbb{Z}_n$ is a surjective function $c : \mathbb{Z}_n \to [r]$. We consider solution sets to the equation $x - y = z^2$, where x, y, and z are elements of $\mathbb{Z}_n$. A solution is a rainbow if each of its elements has a distinct color. In this presentation, we discuss the rainbow numbers, the fewest number of colors needed to guarantee a rainbow solution, of $\mathbb{Z}_n$ for the equation $x-y = z^2$. We find the rainbow number when n is prime, as well as show rainbow-free coloring of some composite numbers n.

Presenter: Paul Fonstad, Franklin College

Getting together apart: Math Club events for uncertain times

This past year forced many of us to turn our in person math club events into virtual ones. Yet while this provided us with new challenges, it also provided us with some unique opportunities to try new activities that can still be used even once events go back to being in person. In this talk, we will examine three events that the Franklin College Math and Computing Club hosted virtually last year and how those events can be used by other schools whether your club meets virtually or in person.

Presenter: Thomas Horine, Indiana University Southeast

Developing an interactive fiction project to engage History of Math students

I have sought ways to encourage students to engage with course material outside of the standard lecture, pencil-and-paper homework, and exam formats. In addition, I have tried to have my students write more about mathematics. In late Summer 2019, I began writing an interactive fiction that incorporated important ideas, procedures, and figures from our History of Mathematics course.

This talk will discuss that process, my short- and long-term aims of the project, and difficulties I have encountered along the way.

Presenter: Ramesh Karki, Indiana University East

Key strategies to engage students in an online math class

Popularity of online education have been growing in recent years. During the current pandemic era, importance of online education has grown more than ever. As the popularity of online education is growing day-to-day, it is a challenging task to engage students in an online mathematics class as it usually has a diverse student body. For effective learning of mathematical content, it is essential to engage students in various course activities. Here we will discuss a few effective strategies used to engage students in an online math class.

Presenter: Michael Karls, Ball State University

Verifying one-dimensional groundwater flow with incomplete data

In 2009 I began a series of student research projects aimed at validating classic groundwater flow models that involve the heat equation. We will look at the following problem, based on the data collected for one of these projects: Suppose you have collected head level data measured at three water wells in a row, but are unsure of the fixed head levels at boundaries to the left and right of these wells. Find a model for this data. We offer two ways to approach this problem which lead to models that produce an excellent match to the data.

Presenter: Sarah Klanderman, Marian University

Joint work with: Patrick Eggleton, Taylor University; Ben Gliesmann, Christian Fenger Academy High School; David Klanderman, Calvin University; and Josh Wilkerson, Regents School of Austin.

Factors that motivate students to learn mathematics

What motivates some students to want to learn mathematics while others do not share similar motivations? Are these factors intrinsic, extrinsic, or a combination of both? To answer these questions, my collaborators [listed above] and I adapted a survey originally developed by Tapia (1996) and later shortened by Lim and Chapman (2015). We administered the survey in multiple middle schools, a high school, and multiple colleges and universities and obtained over 100 completed surveys for each of these educational levels. This presentation offers an analysis of these data as well as comparisons among college participants majoring in mathematics or mathematics education, those majoring in elementary education, and those with a variety of other majors. Come to learn more about why students enjoy learning mathematics and later choose undergraduate majors in the discipline.

Presenter: Naama Lewis, Marian University

Convex optimization in statistics

This presentation will explore the role of convex optimization and the importance of finding solving algorithms for convex problems as it relates to statistics. Many statistical problems can be framed as convex optimization problems. However with the amount of information being collected and analyzed with statistical techniques, the convex problems quickly become unyielding. This makes the development of algorithms that solve large convex optimization problems extremely important. We will explore one such algorithm as well as some potential applications.

Presenter: Richard McHone, Indiana University East undergraduate student
Faculty Advisor: Nayeong Kong, Indiana University East
Convolution inequalities with probability distributions
In probability theory, it is possible to find new probability distributions from addition. For example, to find the sum distribution of two probabilistic random variables, we need to compute the convolution of two given distributions. Unfortunately, this computation is not easy; hence people find inequalities for convolutions in various papers. I focused on one of them. I set out to test a convolution inequality against two existing probability distributions, exponential and normal. Given a specific boundary, our goal was to form a new inequality sufficient for sustaining the terms within the convolution inequality. What was discovered is that neither exponential nor normal distributions were able to hold the given theorem. I further found an upper bound for a constant parameter in the inequality.

Presenter: Lauren M. Nelsen, Colorado College
Trying to make modeling less scary in the differential equations classroom
Many students can be intimidated by open-ended modeling questions, such as those often seen in modeling competitions. For those of us whose work does not regularly involve these types of problems, the modeling process can seem scary when applied to problems that are not neat, clear "textbook" problems. This talk will discuss a humble attempt to tackle some of that scariness in a differential equations class.

Presenter: Hai Phan, DePauw University undergraduate student
Faculty Advisor: Seonguk Kim, DePauw University
Numerical approaches of pricing options in the Cox-Ross-Rubinstein models
I introduce the Cox-Ross-Rubinstein (CRR) model which is used to price European and American Options without complex elements, including dividends, stocks, and stock indexes paying a continuous dividend yield, futures, and currency options. In this talk, I will talk about the CRR model's numerical elements and equations, and a practical event to demonstrate the application of the model in the financial market.

Presenter: Kyle Raihala, Indiana University East undergraduate student
Analyzing the safety of the cycling network in Trier, Germany
Trier is a medium-sized city of approximately $111,500$ people, yet it is nestled in the second most motorized state in the republic, serving as a major cultural hub for the region. With a disjointed patchwork cycling network, cyclists must navigate between sidewalks, sharrows, streets, and various cycle lane forms, some of which simply end, leaving cyclists to deal with sudden changes to their path. Previous research suggests most collisions between cyclists and other vehicles occur at traffic intersections, making them high-priority areas for improvement. We used incident data from the Trier Polizei to identify intersections that are high-risk for cyclists, and we consider common intersection treatments to reduce collisions. In this paper, we use ArcGIS software to analyze incident data and help draw meaningful conclusions that can be used to help city officials with city planning and accident prevention.

Presenter: Jacob Roeder, Trine University undergraduate student[1]

Joint work with: Kayla Barker, Stockton University undergraduate student; Mia DeStefano, Vassar College undergraduate student; Michael Gohn, DeSales University undergraduate student; and Joe Miller, University of Wisconsin La Crosse undergraduate student

Faculty Advisors: Eugene Fiorini, Rutgers University; and Wing Hong Tony Wong, Kutztown University

Two-player pebbling game on a simple graph

A two-player impartial pebbling game is played on a graph made up of vertices and edges connecting the vertices, with some number of pebbles distributed on the vertices. The players take turns removing a fixed number of pebbles a from one vertex and placing a smaller fixed number pebbles b on a vertex that is connected to the original vertex by an edge. Play proceeds in this manner until no more moves can be made, and the last player to make a move wins. The game is said to resolve if there is some number of pebbles m such that the first player to make a move will win the game with m pebbles, regardless of how the pebbles are initially distributed. We looked at pebbling games with the move $a = k+1$ and $b = k$ for an arbitrary integer k on complete graphs (where every vertex is connected to every other vertex by an edge). We determined a formula for the number of pebbles m at which these games on a complete graph with 3 vertices will resolve. We also looked at other variations of pebbling moves and at other kinds of graphs, and we proved that there is no value m so that the pebbling game with the move $a = 2$ and $b = 1$ played on the 4-cycle (4 vertices connected in a closed chain) will resolve, and we have classified whether the first player or the second player will win for different distributions of pebbles in this pebbling game.

[1]Mr. Roeder had to cancel his appearance shortly before the meeting, but during the Zoom meeting, participants were provided with this link to recorded videos of his two talks: `https://drive.google.com/drive/folders/1OqdmqQDOIZw8iEZsSm7QEAvy6Z3m1PnG?usp=sharing`

Presenter: Jacob Roeder, Trine University undergraduate student

Joint work with: Joe Miller, University of Wisconsin La Crosse undergraduate student; Alex Nash, Dickinson College undergraduate student; and Hani Samamah, University of Florida undergraduate student

Faculty Advisors: Brian Kronenthal, Kutztown University; and Wing Hong Tony Wong, Kutztown University

Classifying 2-dimensional real algebraically defined graphs by diameter

A 2-dimensional algebraically defined graph $\Gamma_{\mathcal{R}}(f(X,Y))$ is a bipartite graph, constructed using a ring $\mathcal{R}$ and a bivariate function f, where each partite set is a copy of $\mathcal{R}^2$. In this graph, two vertices (a_1, a_2) and (x_1, x_2) are adjacent if and only if $a_2 + x_2 = f(a_1, x_1)$. The study of algebraically defined graphs can be motivated by incidence geometry, as every graph $\Gamma_{\mathbb{F}_q}(f)$ with girth 6 can be completed to a projective plane of order q.

Previously, all girth 6 graphs $\Gamma_{\mathbb{C}}(f)$ were proved to be isomorphic; however, it was unknown whether all girth 6 graphs $\Gamma_{\mathbb{R}}(f)$ were isomorphic. To address this question, we first prove that whenever p is a polynomial, $\Gamma_{\mathbb{R}}(p)$ has diameter 4 or 5. Moreover, we classify infinite families of such graphs by diameter, including a proof that all known girth 6 graphs $\Gamma_{\mathbb{R}}(p)$ have diameter 4. Ultimately, we use these tools to prove that the two girth 6 graphs $\Gamma_{\mathbb{R}}(XY)$ and $\Gamma_{\mathbb{R}}(X^3Y^3 + XY)$ are non-isomorphic.

Presenter: Alison Rosenblum, Purdue University West Lafayette graduate student

A tourist's guide to o-minimality

Come, step away from the chaos of the real numbers to the idyllic world of o-minimality, where sets and functions behave nicely. Through this expository talk, you will have the opportunity to explore a few notable features of an o-minimal structure, and pay a visit to the cell decomposition theorem. You may even leave with a souvenir proof that all groups are Abelian.