ABSTRACTS OF TALKS PRESENTED TO THE INDIANA SECTION OF THE MAA

1. Introduction

The Fall 2019 meeting of the Indiana Section of the Mathematical Association of America is at Wabash College, October 26. The abstracts appearing here are based on text electronically submitted by the presenters. Contributed talks are listed in alphabetical order by presenter.

2. Invited Talks

Presenter: Jim Daniel, University of Texas at Austin and Treasurer of the MAA

Concave Functions, Jensen's Inequality, Annuities, and Property Settlements at Divorce

This talk first defines concave functions on an interval (pre-calculus required), and then states some of their properties (calculus required to understand part of this). It extends the basic property to derive Jensen's inequality

$$f(E[X]) \geq E[f(X)]$$

for concave functions both algebraically and geometrically (basic probability needed). Finally, it applies these ideas to the present value of an annuity and discusses how this matter arises in valuing pensions for property settlements at divorce.

Presenter: Michael Orrison, Harvey Mudd College

Change of Perspective in Mathematics

Change of perspective is ubiquitous in mathematics. Consider, for example, u-substitution, change of coordinates, Bayes' Theorem, combinatorial proofs, equivalent fractions, isomorphisms, or even the simple fact that $2 + 3 = 3 + 2$. In this talk, I'll offer some reflections on the unifying role that change of perspective plays when we learn, share, create, and discover mathematics. I'll also share how conversations over the years with teachers, undergraduates, and my own children have shaped my understanding of the power of change of perspective.

3. Indiana Project NExT Panel Discussion

Panelists:

- Stacy Hoehn, Franklin College
- Seonguk Kim, DePauw University
- Amber Russell, Butler University

Moderators:

- Livia Hummel, University of Indianapolis
- Ranjan Rohatgi, Saint Mary's College
- Amelia Tebbe, Indiana University Kokomo

Brainstorming Roundtables: Classroom Activities

In this session you will be introduced to in-class activities used by other Section members, as well as invited to share similar ideas, and brainstorm modifications and improvements in a small group setting.

Attendees will be asked to sit in one of three small groups led by a faculty discussion leader. The faculty leaders will introduce an activity they have implemented in the classroom to start a conversation, and moderate the ensuing discussion. During the session, attendees will have the opportunity to join discussions with two of the three faculty leaders.

Stacy Hoehn will share a Desmos activity used in a Real Analysis course. Seonguk Kim will share the Group Study activity as used in a Computational Discrete Mathematics course. Amber Russell will share an Origami Math activity that has been used in an Introduction to Proof course.

4. Contributed Talks

Presenter: Katie Ansaldi, Wabash College

Joint work with: Houssein El Turkey, University of New Haven; Jessica Hamm, Winthrop University; Anisah Nu'Man, Spelman College; Nathan Warnberg, University of Wisconsin-La Crosse; Michael Young, Iowa State College

Rainbow numbers of $\mathbb{Z}_n$ for $a_1x_1 + a_2x_2 + a_3x_3 = b$

An exact r-coloring of a set S is a surjective function $c : S \to [r]$. The rainbow number of a set S for an equation eq is the smallest integer r such that every exact r-coloring of S contains a rainbow solution to eq, that is, a solution in which no two elements have the color. In this presentation, we discuss the rainbow numbers of $\mathbb{Z}_p$, for p prime and the equation $a_1x_1 + a_2x_2 + a_3x_3 = b$. We will also show that some rainbow numbers of $\mathbb{Z}_n$ can be calculated using prime factorizations of n.

Presenter: Tyler Billingsley, Purdue University West Lafayette graduate student

MSC 2010: 14G05

Rational points on conics

Using lots of examples, we will discuss the arithmetic procedure to find rational points on conic sections and the geometric procedure to find all of them. No knowledge of algebraic geometry is assumed. If time permits, we will discuss the limitations of the analogous procedure on elliptic curves.

Presenter: Alden Bradford, Purdue University West Lafayette graduate student

Joint work with: Tarun Yellamraju, Purdue University West Lafayette graduate student

Faculty Advisor: Mireille Boutin, Purdue University West Lafayette

The unexpected virtue of guess-and-check for clustering

How do you find clusters in unlabeled high-dimensional data? There are many, many algorithms which address this problem, but few are as simple as this one, based on repeated random projections onto one dimension. We present the n-TARP clustering algorithm, and explain in part why it works.

Presenter: Joshua Cole, University of Findlay

Joint work with: Peter G. Hinman, University of Michigan emeritus.

Topology and unsolvability

A mathematical problem P is more unsolvable than a mathematical problem Q if from every solution to P a solution to Q can be computed. Computability theorists have long studied relative degrees of unsolvability, especially between sets (or mathematical problems) that are in some sense close to being solvable. A good example is the study of the Turing degrees of computably enumerable sets, such as the halting set. Recently researchers such as Paul Shafer have suggested it is interesting to restrict the mathematical problems under consideration to those of a certain level in the Borel hierarchy. We consider in this talk the relative unsolvability of mathematical problems whose solutions sets are closed in Cantor or Baire Space (in which Shafer is also especially interested). There is a precise sense in which unsolvable problems with closed solution sets are close to being solvable, and this makes the study of their relative degrees of unsolvability especially interesting. Moreover, just as is the case for the study of the computably enumerable sets, the study of the degrees of unsolvability of closed sets sheds light on other topics in computability theory and in the foundations of mathematics. We present work on the question of whether the structure of the degrees of unsolvability for closed sets of Baire Space is the same as for closed sets of Cantor Space.

Presenter: Dennis G. Collins, University of Puerto Rico, Mayagüez (retired)

Toward status-in-life dynamics and drama triangle cycling

Recent work, for example by Leroy Hood, has tried to develop personalized medicine. The paper studies status-in-life (SIL) trajectories versus time based on central forces. Here the SIL scale goes from 0 (powerless and near death) to dependent status (< 4.5) to independent status (> 4.5) to very powerful (near 10). The government central force pulls people down toward 4.5, by taxes for example, if above 4.5, or upward by social programs if below 4.5. The entropy or death central force pulls people toward 0, giving rise to a parabola shape, as in an 1872 Currier & Ives print. In 3-dim. (aggressor, victim, rescuer) SIL space, circular motion can develop, leading to drama triangle cycling.

Presenters: Clarke Criddell and Devin Vanyo, Wabash College undergraduate students

Faculty Advisor: Colin McKinney, Wabash College

MSC 2010: 01A20

Ratios and tangents on Archimedean spirals

Archimedes' geometric research on spirals — like very many ancient mathematical works — depends heavily on ratio theory. Ratio theory provides a means of relating and manipulating magnitudes without ever alluding to quantity. We will first briefly present ratio theory as it was described in Euclid and then show how Archimedes used it to prove a surprising result about tangent lines to Archimedean spirals.

Presenters: Qin David Deng, Image Processing Center, School of Astronautics, Beihang University (Beijing) undergraduate student, and Michael Xue, Vroom Laboratory for Advanced Computing, Indianapolis

Maximizing the final speed of a two-stage rocket using a computer algebra system

A rocket consists of a payload of mass P propelled by two stages of masses m_1 (first stage) and m_2 (second stage), both with structural factor $1-e$. The exhaust speed of the first stage is c_1, and of the second stage c_2. The initial total mass, m_1+m_2 is fixed. The ratio $b = P/(m_1+m_2)$ is assumed very small. According to the multi-stage rocket flight equation ([1]), the final speed of a two-stage rocket is

$$v_f = -c_1 \log(1 - e \cdot m_1/(m_1+m_2+P)) - c_2 \log(1 - e \cdot m_2/(m_2+P)) \tag{1}$$

Let $a = m_2/(m_1+m_2)$, so that (1) becomes

$$v_f = -c_1 \log(1-(e-ea)/(1+b)) - c_2 \log(1 - ea/(a+b)) \tag{2}$$

where $0<a<1$, $b>0$, $0<e<1$, $c_1>0$, $c_2>0$. We seek an appropriate value of a to maximize v_f.

The above rocket performance optimization problem is solved with the help of a Computer Algebra System (CAS) ([2]). We found that the value of a for a maximum final speed is

$$\sqrt{c_2 b/c_1} + O(b) \tag{3}$$

and the maximum speed is

$$-(c_1+c_2)\log(1-e) - 2e\sqrt{c_1 c_2 b}/(1-e) + O(b). \tag{4}$$

We compute the first order Taylor series of a and v_f in b to replace their complex expressions respectively. Results produced by the CAS are validated extensively through rigorous mathematical analysis.

References

[1] M. Xue, *Viva Rocketry! Part 2*
`http://vroomlab.wordpress.com/2019/01/31/viva-rocketry-part-2-2`

[2] *Omega: A Computer Algebra System Explorer*
`http://www.omega-math.com`

Presenter: Dylan Harker, Franklin College undergraduate student

Determinants of X-Matrices

An X-Matrix is any square matrix whose only nonzero entries are on its diagonal and its counter diagonal. We know how to quickly find the determinant of any triangular matrix as the product of its diagonal entries, but what about the case of the X-Matrix? We will explore the pattern of the determinants of X-Matrices, develop a conjecture, and prove a formula for this calculation.

Presenter: Joshua Holden, Rose-Hulman Institute of Technology

Joint work with: Soully Abas, Rose-Hulman Department of Humanities, Social Sciences, and Arts

MSC 2010: 97M80

Between the Two Cultures: Teaching math and art to engineers (and scientists and mathematicians)

C. P. Snow famously categorized modern intellectual life as being split between the culture of the sciences and the culture of the humanities, and said that solving the world's problems requires bringing these two cultures together. Math and art classes inherently try to do that. However, most math and art classes we have heard of focus primarily on introducing liberal arts students to mathematics. In the Spring Quarter of 2019 Soully Abas and I team-taught a different sort of course in mathematics and art at the Rose-Hulman Institute of Technology. Rose-Hulman is a private, undergraduate-focused institution, all of whose students major in engineering (predominantly), science, or mathematics. We wanted to build on the mathematical knowledge that our STEM students already have with a focus on having them make physical art objects. Soully is an art professor specializing in oil painting and printmaking. Josh is a math professor interested in the mathematics of fiber arts such as embroidery, knitting, crochet, and weaving. Our goal was for students to use their existing mathematical (and perhaps artistic) knowledge to reinforce new artistic (and perhaps mathematical) experiences. Ideally, the knowledge and experience gained will increase their appreciation for both the beauty of mathematics and the importance of art and help them prepare for productive lives solving the world's problems.

Presenter: Michael Karls, Ball State University

Designing an environmental modeling course

We will look at the challenges of designing and implementing an environmental modeling course for a general audience; provide an overview of topics covered and resources used; give examples of homework problems, exam questions, and project topics chosen for the course; address issues that have arisen.

Presenter: Nayeong Kong, Indiana University East

Random geometric graphs

This talk has two parts. In the first part, we focus on Random Inner Product Kernel Matrices. There are many research papers that have proved that the limiting empirical spectral distribution (ESD) of such matrices A converges to the Marchenko-Pastur distribution. Among them, we will consider one of the important results.

The second part is about Random Geometric Graphs. The theory of random graphs was founded in the late 1950s by Erdős and Rényi (1959). The work of Watts and Strogatz (1998) and Barabási and Albert (1999) at the end of the last century initiated new interest in this field. The subject is at the intersection between graph theory and probability theory.

In the second part, Random Geometric Graphs on the Unit Sphere are considered. Observing that adjacency matrices of these graphs can be thought of as random inner product matrices, we are able to use an idea of Cheng-Singer to establish the limit for the ESD of these adjacency matrices.

Presenter: Rodney Lynch, Indiana University - Purdue University Columbus

New Venn diagram puzzles

Venn diagram problems in which the information is presented in a visual format will be presented. Puzzles will be created by considering projections of a Venn diagram onto various subdiagrams. I will also show how much of a Venn diagram can be obtained by considering only these projections.

Presenters: Lauren M. Nelsen and Luke L. Nelsen, University of Indianapolis

Joint work with: Kirk Boyer, Google; Florian Pfender, University of Colorado Denver; Elizabeth Reiland, AwesomeMath; and Ryan Solava, St. Mary's College

Ups and Downs: A two-player game with dots

In 1935, Erdős and Szekeres proved that $(m-1)(k-1)+1$ is the minimum number of points in the plane which definitely contain an increasing subset of m points or a decreasing subset of k points (as ordered by their x-coordinates). We consider their result from an on-line game perspective. In this talk, we discuss the original result of Erdős and Szekeres and introduce the new game and its known results.

Presenter: Christina Pospisil, University of Massachusetts Boston undergraduate student

Generalization Theory of Linear Algebra I: An embedding algorithm and an appropriate inverse for non-injective mappings in one dimension

An algorithm for multiplying and adding matrices regardless of dimensions via an embedding is presented. An equivalent embedding for a general determinant theory is also investigated. Appropriate inverses for non-injective mappings in one dimension are presented. Future work will explore applications to physics and other natural sciences.

Presenter: Marshall Whittlesey, California State University San Marcos
MSC 2010: 51M
A course in spherical geometry for math majors
A century ago, spherical geometry was a standard part of the mathematics curriculum in high schools and colleges. Today most mathematicians only learn about it as a short topic in geometry survey courses. In this talk we explore the idea of teaching spherical geometry to mathematics majors at greater depth by discussing some key theorems of spherical geometry, short proofs, and applications to other areas such as astronomy, crystallography, and polyhedra. We discuss the use of different techniques (synthetic versus analytic) to advantage in this subject in the hope that the student will benefit from thinking about when each method is appropriate. In particular, the quaternion algebra turns out to be a useful tool for doing geometry on the sphere. We also think that comparison of theorems of spherical geometry to those of plane geometry is a good way for the student to see in a tangible way how changing axioms results in different theorems. We think that exposure to spherical geometry is particularly good for future high school teachers, but also that more mathematicians should be aware of its theorems and applications. The approach discussed in this talk is taken in my new book *Spherical Geometry and its Applications*, available from CRC Press.

Presenter: Godfred Yamoah, Trine University
A coupling of time and space adaption schemes for Richards' equation
Obtaining robust and efficient numerical solutions for fluid flows in porous media continues to be challenging, in particular for infiltration into non-uniform porous media. In this work, we present a Galerkin finite element method that coarsens and refines the mesh based on an error indicator paired with a temporal adaption scheme that controls the local truncation error at each time step. The temporal scheme is based on linear extrapolation for smooth functions, but has been proven to work for non-smooth problems when a finite-difference Jacobian is used. We present numerical results for the pressure head form of Richards' equation for infiltration problems, which is a non-smooth model. We provide error and work measures that compare the performance of the joint spatial-temporal adaption scheme to that of a fixed grid approach with temporal error control scheme, a fixed grid approach with heuristic time stepping method, and a spatially adaptive approach with heuristic time stepping.

Presenter: Yufei Zhang, Saint Mary's College undergraduate student
Faculty Advisor: Ranjan Rohatgi, Saint Mary's College
MSC 2010: 05C78
Total difference labelings of graphs
Inspired by graceful and total labeling of graphs, we introduce the idea of total difference labelings. A total difference labeling of a graph G can be obtained by labeling the vertices from a set of positive integers and labeling the edges by taking the absolute difference of the end vertices, and then requiring that the labels form a total labeling of G. The minimum possible greatest label on G is called the total difference chromatic number. In this talk, we will determine the total difference chromatic number of paths, cycles, stars, and caterpillars.