ABSTRACTS OF TALKS PRESENTED TO THE INDIANA SECTION OF THE MAA

1. Introduction

The Fall 2018 meeting of the Indiana Section of the Mathematical Association of America is at Hanover College, October 13. The abstracts appearing here are based on text electronically submitted by the presenters. Contributed talks are listed in alphabetical order by presenter.

2. Invited Talks

Presenter: Jacqueline Jensen-Vallin, Lamar University and Editor of *MAA Focus*

Let's Get Knotty

My early interest in numbers and patterns led me down a (nonlinear) path to mathematics, which has led me to the twisty world of knots. Mathematically, knots are non-intersecting closed curves in space. We will use sequences and patterns to explore this world and play with a classic question in knot theory — given a knot diagram, how do I identify the knot? There will be plenty of examples, conjectures, and fun!

Presenter: Emilie Purvine, Pacific Northwest National Laboratory

How Can Mathematicians Help Advance Cyber Security?

The world is getting more interconnected every day. New devices are providing near-constant connectivity for all aspects of our lives. There are devices that we put in our homes like smart thermostats, and ones that we wear on our bodies like fitness trackers or smart watches. There are even wireless enabled medical devices that can be implanted inside our bodies. Our cars and phones also keep us on the grid when we're on the go. It is increasingly difficult to become disconnected in today's society. Together this means that there are even more opportunities to be hacked, and with much higher consequences. Cyber security experts, those with backgrounds in areas like computer science and ethical hacking, are constantly working to monitor and secure these systems and networks. There are vast databases of known attack signatures that can be checked against current behaviors to identify potentially malicious activity. But adversaries are constantly changing their tactics, techniques, and procedures to evade these signature-based detection strategies. This is where mathematicians are helping to advance the field. Cyber systems, when abstracted, look a lot like a mathematical object called a graph. Studying the statistical and topological properties of this graph can help us to quantify when large unexpected changes are being made to the system. Additionally, by understanding which graph properties contribute to weaknesses in current systems we can design stronger ones for the future. In this talk, Dr. Purvine will survey the relevant cyber landscape, introduce mathematical models of cyber networks using discrete mathematics, graph theory, and topology, and provide some answers the question asked in the title: "how can mathematicians help advance cyber security?"

Applications of Topology for Information Fusion *

In the era of "big data" we are often overloaded with information from a variety of sources. Information fusion is important when different data sources provide information about the same phenomena. For example, news articles and social media feeds may both be providing information about current events. In order to discover a consistent world view, or a set of competing world views, we must understand how to aggregate, or "fuse", information from these different sources. In practice much of information fusion is done on an ad hoc basis, when given two or more specific data sources to fuse. For example, fusing two video feeds which have overlapping fields of view may involve coordinate transforms; merging GPS data with textual data may involve natural language processing to find locations in the text data and then projecting both sources onto a map visualization. But how does one do this in general? It turns out that the mathematics of sheaf theory, a domain within algebraic topology, provides a canonical and provably necessary language and methodology for general information fusion. In this talk I will motivate the introduction of sheaf theory through the lens of information fusion examples.

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3. Indiana Project NExT Panel Discussion

Panelists:

- Emilie Purvine, Pacific Northwest National Laboratory
- Joseph Eichholz, Rose-Hulman Institute of Technology
- Lisa Holden, Northern Kentucky University
- Dhanuja Kasturiratna, Northern Kentucky University

Moderator: Amber Russell, Butler University

Preparing Students for BIG Careers

How can faculty work to best prepare students for jobs in Business, Industry, and Government (BIG)? Our panelists share their suggestions from a variety of different perspectives. Employed at a national laboratory, Emilie Purvine has first hand experience of the skills students must develop to be successful. The remaining panelists have all participated in the PIC Math program (Preparation for Industrial Careers in Mathematical Sciences) sponsored by the MAA. Lisa Holden and Dhanuja Kasturiratna worked on a project with American Modern Insurance Group, while Joseph Eichholz worked with Christina Selby (now at Johns Hopkins University) on two projects: one with Crane Payment Innovations, the other with 84.51 on a project related to direct marketing.

4. Contributed Talks

Presenter: Axel Brandt, Northern Kentucky University
MSC 2010: 05C15
Polynomial peacemakers: avoiding scheduling conflicts
We discuss two questions related to scheduling committee meetings for the U.S. House of Representatives so that no representative is conflicted by trying to be in two meetings at the same time. These questions will be approached by constructing a polynomial for which every committee schedule resulting in a conflict corresponds to a zero of the polynomial. After providing some intuition, we will discuss the Combinatorial Nullstellensatz and how it can be used to guarantee the existence of a committee schedule without conflicts.

Presenter: Max Cartor, Bellarmine University undergraduate student
Faculty Advisor: Greg Kelsey, Bellarmine University
The topology of the unbased visual boundary of the Diestel Leader graph
We will examine the topology of the unbased visual boundary of the Diestel Leader graph. To do so, we will first establish a background on basic elements of topology, based and unbased visual boundaries, the Diestel Leader graph, and the lamplighter group. We then focus on utilizing geodesic rays to examine the topology of the unbased visual boundary. Manipulation of the starting position of geodesic rays in this space will show that the boundary has the trivial topology.

Presenter: Katie Christensen, University of Louisville graduate student
Faculty Advisor: Hamid Kulosman, University of Louisville
Algebraic properties of neural codes
The neural rings and ideals as an algebraic tool for analyzing the intrinsic structure of neural codes were introduced by C. Curto et al. in 2013. Since then they were investigated in several papers, including the 2017 paper by Gunturkun et al., in which the notion of polarization of neural ideals was introduced. In this paper we extend their ideas by introducing the notions of polarization of motifs and neural codes. We show that the notions that we introduced have very nice properties which could allow the studying of the intrinsic structure of neural codes of length n via the square free monomial ideals in $2n$ variables and interpreting the results back in the original neural code ambient space.

Presenter: Casey Garner, Rose-Hulman Institute of Technology undergraduate student

Joint work with: Qianli Song, University of Hong Kong undergraduate student; Antonio Farah, University of Texas at Austin undergraduate student; and Adam Dhillon, Harvey Mudd College undergraduate student

Faculty Advisor: Alexander Vladimirsky, Cornell University

Initial uncertainty: lurking in the shadows

Many times a path is desired from one location to another that optimizes some quantity such as time, safety, or energy. This presentation will discuss how to determine the path for an evader in the presence of a single stationary observer. The evader is attempting to make it from a starting point to a target along the path that minimizes its expected observability, or likelihood of being viewed. The evader knows the possible locations for the observer and the probability distribution over them. The map on which the evader is travelling contains obstacles that the observer cannot see around. Thus, shadow regions exist where the evader cannot be viewed by any of the potential observer locations. When the evader travels outside of the shadows, it gains information about the location of the observer and uses this to improve the path it is taking. Our approach involved analyzing the Eikonal PDE, for which we developed algorithms to improve computational efficiency. This was an REU project done at Cornell this past summer with three other students. No previous knowledge of the Eikonal equation or computational methods is necessary.

Presenter: Dylan Green, Trevecca Nazarene University undergraduate student

Joint work with: Grant Fickes, Kutztown University of Pennsylvania; Karen McCready, King's College; Kathleen Ryan, DeSales University; Nathaniel Sauerberg, Carleton College; Jill Stifano, Fairfield University

Characterizing bounds of proper diameter in 2-connected graphs

A properly colored path is a path in which no two consecutive edges have the same color. A properly connected coloring of a graph is one in which there exists a properly colored path between every pair of vertices. Given a graph G with a properly connected coloring c, the proper distance between any two vertices is the length of a shortest properly colored path between them. Furthermore, the proper diameter of G is the largest proper distance between any pair of vertices in G under the given coloring c. Since there can be many properly connected colorings of G, there are possibly many different values for the proper diameter of G.

If G has n vertices, a natural upper bound for its proper diameter is $n-1$, but this value is not attainable for all graphs, such as graphs without a Hamiltonian path. We completely characterize all 2-connected graphs in which the upper bound of $n-1$ is achievable with 2 colors. To do so, we introduce a new family of graphs called tau graphs, and we show that a 2-connected graph on n vertices with a properly connected 2-coloring has a maximum proper diameter of $n-1$ if and only if the graph is a tau graph.

Presenters: Zach Hollis and Dylan Kunce, Trine University undergraduate students

MSC 2010: 60

When do you call the bullpen?

A baseball team has two goals: to avoid making outs on offense; and force the other team into making outs on defense. One can think of an opponent successfully reaching base as a temporary setback to achieving this second goal. We use Markov chains to model such processes that allow for temporary setbacks before reaching absorbing success or fail states and apply this model to baseball and use it to analyze current trends in the usage of starting pitchers. Our model can be easily modified to apply to applications such as certain platform video games, the completion of sequential courses (Calculus I $\rightarrow$ Calculus II $\rightarrow$ Calculus III; for instance), and other order specific tasks.

Presenter: Seonguk Kim, Depauw University

Matrix representations for differential operators

In this talk, I will suggest a basic idea about how to make a matrix representation for the one-dimensional differential operator $D = \dfrac{d^2}{dx^2}$. To do so, I first introduce the basic concept of matrix representation for linear operators which we have learned in a Linear Algebra course. Next, I will introduce a functional space called Hilbert space, an inner-product complete space. This space plays a crucial role in transferring from functions to infinite dimensional coordinate systems. Finally, I will give an idea about thinking of the differential operator as a linear operator and trying to express the operator as an infinity by infinity matrix.

Presenter: Rodney Lynch, Indiana University - Purdue University Columbus

A construction that gives a group of non-invertible matrices

Recall that the adjoint of a square matrix is the transpose of its matrix of cofactors. Let $\mathbf{b} = \begin{pmatrix} b_1 & b_2 & \cdots & b_n \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} c_1 & c_2 & \cdots & c_n \end{pmatrix}$ be row matrices with entries in a field k (which you can take to be the real numbers for this talk). Also assume $\mathbf{b}\mathbf{c}^T = 1$ (usual dot product). Note that $\mathbf{b}^T\mathbf{c}$ is a rank one matrix, hence is non-invertible if $n \geq 2$.

I will show that the collection of non-invertible matrices which satisfy $\operatorname{adj}(A) = \mathbf{b}^T\mathbf{c}$ will always form a group, and give its identity and the formula for the inverse of an element. Note that the identity cannot be the identity matrix!

This is the second part of the talk that I gave at the Spring meeting of the MAA at Valparaiso University. In the first part I gave the description of all such matrices A satisfying $\operatorname{adj}(A) = \mathbf{b}^T\mathbf{c}$. I will give this description again but will not need its derivation, so it is not necessary to have seen my first talk.

Presenter: Prayat Poudel, Centre College
Joint work with: Tim Ablondi and Yifan Ye, Centre College
MSC 2010: 57M25
Invariants of two-bridge links
Two-bridge knots and links, also known as rational knots/links, are a class of knots/links with particularly nice geometry. They are also of fundamental importance in the study of DNA recombination. In this talk, we will define 2-bridge links, mention some of their geometric properties, and discuss recent work in computing some classical invariants of these links.

Presenter: Ranjan Rohatgi, Saint Mary's College
Joint work with: Aysel Erey, Gebze Technical University, Turkey; Zachary Gershkoff, Louisiana State University; and Amanda Lohss, Messiah College
MSC 2010: 05C75
Graphs on a train
In a k-regular graph, each vertex has degree k. A permutation graph has as its vertices the elements of a permutation, and two vertices are connected by an edge if the corresponding elements are reversed in the permutation. In this talk we characterize graphs that are both 3-regular and permutation, and then count the number of 3-regular permutation graphs on a fixed number of vertices.

Presenter: Michael Xue, Vroom Laboratory for Advanced Computing
A non-iterative method for solving nonlinear equations
The Newton-Raphson method is the most commonly used iterative method for finding the root(s) of a real-valued function or nonlinear systems of equations. However, its convergence is often sensitive to the error in its initial estimation of the root(s). This talk will present a non-iterative method that mitigates non-convergence. An auxiliary initial-value problem of ordinary differential equation(s) is generated by a Computer Algebra System first, then integrated numerically over a closed interval. The solution(s) to the original systems of nonlinear equations is obtained non-iteratively at the end of the interval. A proof of the theorem serving as the base for this new method is presented at the talk. Several examples will illustrate its guaranteed convergence, a clear advantage over the Newton-Raphson method.

Presenter: Jihyeon Jessie Yang, Marian University - Indianapolis
Combinatorics for geometry: tropical fan and Newton-Okounkov body
In this talk, I will introduce two new branches in mathematics that are actively being developed: Tropical Geometry and Newton-Okounkov-Body Theory. One of many motivations for these developments is to understand geometrical problems in terms of combinatorics. There are many interesting problems. Especially, there are subtle relations between these two approaches. Starting from a simple matrix, I will show how to construct two key notions in these fields.